THE LOCATION OF THE ZEROS OF POLYNOMIALS WITH COMPLEX COEFFICIENTS

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1. **Introduction.** In a recent paper *On the zeros of polynomials with complex coefficients* [2],[1] we gave an algorithm for determining the number of zeros in any half-plane. We here extend these methods to the actual computation of the zeros by successive approximation. The computation involves only the repetitive processes of synthetic division and cross-multiplication, and *computation with complex numbers has been eliminated.* This method is therefore especially suitable where the computing is to be done by a clerical staff with the use of ordinary calculating machines.

While relatively few methods for finding the zeros of complex polynomials have been given, there are many methods for real polynomials. Runge [12], in 1898, made a comprehensive survey of known processes, and recently this summary was brought up to date by Fry [3]. Among the various methods, Lagrange [10] was one of the first to use continued fractions in the solution. On the other hand, at Bell Telephone Laboratories, the solution of the problem has been reduced to a mechanical one by means of the isograph [1], a machine which solves real polynomials up to the tenth degree.

The method described in this paper is similar to that of G. R. Stibitz [3], who divides the complex plane into horizontal and vertical strips, and to that of F. L. Hitchcock [5], who also uses successive approximation and the greatest common divisor process, but entirely by means of rectangular coordinates. Hitchcock's process consists of Horner's method to approximate the real part of the zero and the euclidean algorithm for the greatest common divisor to find the imaginary part. However, the computational routine is relatively complicated compared with the simple processes in the method given below. This simplification is partly due to the formulas given in §5 for the continued fraction expansion of a rational function.

2. **The method.** (i) *The separation of the zeros into annular rings.* After bounds for the moduli of the zeros of a polynomial have been obtained (see §6), the next step in the computation is the separation of the zeros into different annular rings. In this way, closer bounds for the moduli of the individual zeros are found.

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[1] Numbers in brackers refer to the bibliography at the end of the paper.