play than the classical checker game. The checkers family can also be extended to spaces of higher dimensions. (Received July 12, 1946.)

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326. H. W. Becker: *Stirling's numbers of the third kind.*

These are defined by ${}_{c\pm}^{x}S_{\pm r+1}=(x\pm c\pm r)\cdot{}_{c\pm}^{x}S_{\pm r}+{}_{c-1\pm}S_{\pm r}$, in which the weighting coefficient combines the characteristics of the recurrences for the Stirling's numbers of the first and second kinds. However, tables of the four varieties $\pm\pm$ are not only matrix products of certain tables of the first two kinds, but also unexpectedly simple vector multiples of the table of binomial coefficients. Summed over c, these are expressible by polynomials which are Chrystal-Jordan factorials of Stirling's polynomials of the second kind, and have the remarkable property of a triple recurrence. For example, ${}_{+}^{x}S_{-r+1}=(1+(x))^{r+1}=({}_{+}S+x)_{r+1}={}_{+}^{x}S_{-r}+x\cdot{}_{+}^{x-1}S_{-r}={}_{+}^{x-1}S_{-r+1}+(r+1)\cdot{}_{+}^{x-1}S_{-r}={}_{+}^{x+1}S_{-r}+(x-r)\cdot{}_{+}^{x}S_{-r}$. The combinatorial interpretations are rather intricate in terms of either permutations or rhyme schemes. (Received July 15, 1946.)

327. David Blackwell: *Conditional expectation and unbiased sequential estimation.*

It is shown that $E[f(x_\alpha)E_\alpha y]=E(fy)$ whenever $E(fy)$ is finite, and that $\sigma^2(E_\alpha y)\leqq\sigma^2(y)$, with equality holding only if $E_\alpha y=y$, where $E_\alpha y$ denotes the conditional expectation of y with respect to the family of chance variables x_α. These results imply that whenever there is a sufficient statistic u and an unbiased estimate t, not a function of u only, for a parameter p, the function $E_u t$, which is a function of u only, is an unbiased estimate for p with variance smaller than that of t. A sequential unbiased estimate for a parameter is obtained, such that when the sequential test terminates after i observations, the estimate is a function of a sufficient statistic for the parameter with respect to these observations. A special case of this estimate is that obtained by Girshick, Mosteller, and Savage (Ann. Math. Statist. vol. 17 (1946) pp. 13–23) for the parameter of a binomial distribution. (Received July 5, 1946.)

328. Paul Boschan: *The consolidated Doolittle technique.*

The quadratic matrix notation is interpreted as a segment in a sequence of matrices wherein each successor matrix is augmented by a bordering row and column. Extension theorems based on this idea date back into the last century. The step from the original concept to one of higher order is also fruitful in discussing inverse matrices, specifically the inverse of a symmetric matrix. The symmetry of the matrix of normal equations for a set of multiple regression-coefficients is restored by adding the transpose of the column on the right side of the equations, that is, the co-variances with the dependent variable and the variance of the dependent variable itself. The inverse of this matrix can be constructed as a partial sum over a series of matrices. Each individual element of this series is in itself meaningful. The solution for the set of multiple regression coefficients relating the kth variable to the preceding $(k-1)$ variables is a column matrix. The product of this matrix with its transpose expressed in terms of the residual variance forms the kth term in the matrix series. The summation of the first n products yields the inverse matrix. This characteristic of the inverse can be used to great advantage in the standardization of elementary computational steps. (Received July 17, 1946.)