VECTOR FIELDS AND RICCI CURVATURE

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We shall prove theorems on nonexistence of certain types of vector fields on a compact manifold with a positive definite Riemannian metric whose Ricci curvature[1] is either everywhere positive or everywhere negative. Actually we shall have some relaxations of the requirements both as to curvature and as to compactness. We shall deal with real spaces with a customary metric and with complex analytic spaces with an Hermitian metric. In the latter case we shall impose on the metric a certain restriction, first explicitly stated by E. Kaehler,[2] which will be quite indispensable to our argument. In order to elucidate the rôle of this restriction we shall include a systematic introduction to the theory of Hermitian metric.

For positive curvature we shall have the theorem that on a compact space there exists no vector field for which the *divergence* and *curl* both vanish. In the complex case there exists no vector field whatsoever whose covariant components are analytic functions in the complex parameters. If we only assume that the curvature is nonnegative, then there are some "exceptional" vector fields in directions of spatial flatness. A principal result will be the following theorem on meromorphic functions. If a complex space with positive curvature is covered by a finite number of neighborhoods, if a meromorphic functional element is defined in each neighborhood, and if the difference of any two meromorphic elements is holomorphic wherever the elements overlap, then there exists *one* meromorphic function on the space which differs by a holomorphic function from each meromorphic element given. In a previous paper[3] this conclusion was drawn in the

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[1] Also called mean curvature; the definition will be restated later in the text. Interesting facts relating Ricci curvature to Riemann curvature have been given by T. Y. Thomas, *On the variation of curvature in Riemann spaces of constant mean curvature*, Annali di Matematica Pura ed Applicata (4) vol. 13 (1935) pp. 227–238, and *New theorems on Riemann-Einstein spaces*, Rec. Math. (Mat. Sbornik) N.S. vol. 3 (1938) pp. 331–340. Also, for the application of the Laplacean on compact Riemannian spaces see T. Y. Thomas, *Some applications of Green's theorem for compact Riemann spaces*, Tôhoku Math. J. vol. 46 (1940) pp. 261–266.

[2] *Ueber eine bemerkenswerte Hermitische Metrik*, Abh. Math. Sem. Hamburgischen Univ. vol. 9 (1933) pp. 173–186; see also S. Chern, *Characteristic classes of Hermitian manifolds*, Ann. of Math. vol. 47 (1946) pp. 85–121, especially pp. 109–112.

[3] S. Bochner, *Analytic and meromorphic continuation by means of Green's formula*, Ann. of Math. vol. 44 (1943) pp. 652–673, especially p. 672, Theorem 15.