A SUFFICIENCY THEOREM FOR DIFFERENTIAL SYSTEMS

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1. **Introduction.** This paper is concerned with a boundary value problem involving differential equations and boundary conditions of the form

$$y_i' = [A_{ij}(x) + \lambda B_{ij}(x)]y_j,$$

$$(1.1)\quad s_i[y, \lambda] \equiv (M_{ij}^0 + \lambda M_{ij}^1)y_j(a) + (N_{ij}^0 + \lambda N_{ij}^1)y_j(b) = 0 \qquad (a \leqq x \leqq b;\ i, j = 1, 2, \cdots, n),$$

where the matrix of constants $\|M_{ij}{}^0+\lambda M_{ij}{}^1, N_{ij}{}^0+\lambda N_{ij}{}^1\|$ has rank n for all values of the characteristic parameter λ. In his dissertation the author [3][1] extended to such systems the concept of definite self-adjointness introduced by Bliss [2] for problems with boundary conditions independent of the parameter. Earlier, Bliss [1] had formulated a definition of definite self-adjoint systems in such a manner that systems of this type had infinitely many characteristic values. This property is in general no longer true for systems that are definitely self-adjoint in the modified sense of Bliss [2], and the analogous definition of Bobonis [3] is such that definitely self-adjoint systems (1.1) need not possess an infinitude of characteristic values. As shown in [3], however, for definitely self-adjoint systems (1.1) the characteristic values are all real and have indices equal to their multiplicities; moreover, such systems admit expansion theorems analogous to those obtained by Bliss [2].

It is the purpose of the present paper to consider a definitely self-adjoint system (1.1) which satisfies the additional condition that the matrix $\|B_{ij}(x)\|$ is of constant rank on the interval $a \leqq x \leqq b$. Such a system is shown to be equivalent to a boundary value problem associated with the second variation of a calculus of variations problem of the type considered by Reid [4], and the extremizing properties of the characteristic values of the equivalent problem lead to necessary and sufficient conditions for the given problem to have an infinitude of characteristic values. The methods of proof herein used are analogous to those employed by Reid [5] in establishing the corresponding results for definitely self-adjoint systems whose boundary conditions are independent of λ.

Presented to the Society, April 29, 1944; received by the editors January 22, 1945, and, in revised form, August 29, 1945, and November 19, 1945.

[1] The numbers in brackets refer to the bibliography at the end of this paper.