

POLYNOMIAL DOMINANTS

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Let $f(x)$ be a continuous function of period 2π , and let $F_n(x)$ be a trigonometric polynomial of degree n which dominates $f(x)$ at the points $x_j^n = 2j\pi/(2n+1)$ where $j=0, \dots, 2n$, that is, $F_n(x_j^n) \geq |f(x_j^n)|$. The present paper is concerned with relations of inequality between various integral means of the functions $f(x)$ and $F_n(x)$. A similar problem was considered by Hardy and Littlewood [2]¹ under the hypothesis that the Fourier series of a function $F(x)$ was a majorant of the Fourier series of $f(x)$. A further variation concerning the means of the Fourier coefficients under the hypothesis of functional dominance, that is, $F(x) \geq |f(x)|$, was treated by the author [1]. In the present work, heavy reliance is made on the results of Marcinkiewicz and Zygmund in their paper *Mean values of trigonometric polynomials* [4]. This paper will be referred to in the sequel as MV.

The results which are obtained are of the type

$$(1) \quad M_a(f) \leq A M_b(F_n) + B\omega(2\pi/n),$$

where (i) $M_a(g)$ represents some integral mean of $g(x)$; (ii) A and B are constants independent of n and the functions $f(x)$ and $F_n(x)$, but dependent on the particular mean M_a ; and (iii) $\omega(\delta)$ is the modulus of continuity of $f(x)$.

It can easily be seen that neither term on the right of inequality (1) can be omitted. Suppose first that, for some n , $f(x_j^n) = 0$ but $f(x)$ is not identically zero, and also that $F_n(x_j^n) = 0$ so that $F_n(x) = 0$. Then the left side of (1) will not be zero while the first term on the right vanishes, and consequently the second term on the right is essential. Finally, if the first term were not present in the right member of (1), the left side would be different from zero for non-null functions while the right side would tend to zero as $n \rightarrow \infty$.

Throughout the remainder of the paper we shall suppose that $x_j^n = 2j\pi/(2n+1)$; that $\phi_{2n+1}(t) = x_j$ when $x_j \leq t < x_{j+1}$; and that a letter A, B, \dots with subscript will denote a constant depending only on those parameters appearing in the subscript. The same letter may have a different value at each appearance.

THEOREM 1. For $1 < r < \infty$,

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¹ Numbers in brackets refer to the references cited at the end of the paper.