APPROXIMATION IN THE SENSE OF LEAST pTH POWERS WITH A SINGLE AUXILIARY CONDITION OF INTERPOLATION

A. SPITZBART

Introduction. Let $w = g(z)$ map the interior of the analytic Jordan curve C conformally into the interior of the circle $|w| = 1$. We shall say that the function $f(z)$, analytic interior to C, is of class E_p there if $\int_{C_r} |f(z)|^p |dz|$ is bounded for $r < 1$, where C_r is the curve $|g(z)| = r$. A function is of class H_p if it is analytic for $|z| < 1$, and of class E_p there. We are taking $p > 0$.

Of the functions $f(z)$, analytic interior to C, of class E_p there, with $f(\alpha) = A$ ($z = \alpha$ a point interior to C, A an arbitrary constant), let $F_0(z)$ be the one[1] which minimizes the integral $\int_C |f(z)|^p |dz|$. Let $P_n(z)$ be the minimizing polynomial of degree n with $P_n(\alpha) = A$, for this integral. We shall prove that the sequence $P_n(z)$, $n = 0, 1, 2, \cdots$, *converges maximally* to $F_0(z)$ on the closed set Γ, consisting of C and its interior, and then derive some extensions. We use the term *maximal convergence* in the sense of J. L. Walsh [3, p. 80].[2]

We denote by ρ the maximum value of R such that the minimizing function can be extended so as to be analytic and single-valued interior to Γ_R, as used by Walsh [3, p. 80].

1. **Inequalities: unit circle.** We shall start with the results for the unit circle.

THEOREM 1.1. *Of the functions $f(z)$ of class H_p ($p > 0$) interior to C: $|z| = 1$, with $f(\alpha) = A$, $|\alpha| < 1$, the one which minimizes the integral $\int_C |f(z)|^p |dz|$ is given by $F_0(z) = A[(|\alpha|^2 - 1)/(\bar{\alpha}z - 1)]^{2/p}$, with the branch for which $F_0(\alpha) = A$.*

For, it is true that $\int_C |f(z)|^p |dz| \geqq 2\pi |f(0)|^p$, so that for the case $\alpha = 0$, the minimizing function is $F_0(z) = A$. If, in the general case, we map the interior of the unit circle conformally into itself, with $z = \alpha$ corresponding to the origin, the desired result is obtained.

The main new tool exhibited by the paper is the following theorem.

THEOREM 1.2. *Let $f_n(z)$ be a sequence of functions of class H_p with $f_n(\alpha) = A$ and $\int_C |f_n(z)|^p |dz| \leqq \int_C |F_0(z)|^p |dz| + \epsilon_n$, where $\epsilon_n \to 0$ as*

Presented to the Society, January 1, 1941; received by the editors December 8, 1945.

[1] The minimizing function $F_0(z)$ is unique.

[2] Numbers in brackets refer to the references cited at the end of the paper.