

## ABSTRACTS OF PAPERS

### SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

### ALGEBRA AND THEORY OF NUMBERS

#### 53. A. A. Albert: *On Jordan algebras of linear transformations.*

Consider any linear space  $A$  of linear transformations  $a, b, \dots$  over a field  $F$  of characteristic not two such that  $a \cdot b = 1/2(ab + ba)$  is in  $A$  for every  $a$  and  $b$  of  $A$  where  $ab$  is the associative product. Then  $A$  is called a *Jordan algebra* with respect to the operation  $a \cdot b$ . The radical of  $A$  is defined to be its maximal solvable ideal  $N$  and it is shown to be solvable if and only if all transformations in  $A$  are nilpotent. A trace criterion is derived in case  $F$  is nonmodular, and a Pierce decomposition relative to an idempotent is obtained. The trace condition is then used to derive the analogues of the associative algebra theorem in the case of a principal idempotent. If  $N=0$ ,  $A$  is called semisimple and it is shown that  $A$  is a direct sum of simple algebras. All simple Jordan algebras over an algebraically closed field are obtained and the results may be combined with those of a Chicago dissertation of G. Kalisch to yield all simple Jordan algebras over any nonmodular field. (Received December 21, 1945.)

#### 54. B. L. Brown and N. H. McCoy: *Radicals and subdirect sums.*

Suppose that in each ring  $\mathfrak{R}$  there is assigned a mapping  $F$  of  $\mathfrak{R}$  into the set of two-sided ideals of  $\mathfrak{R}$  in such a way that if  $a \rightarrow \bar{a}$  is a homomorphism of  $\mathfrak{R}$  on  $\bar{\mathfrak{R}}$ , then  $F(\bar{a}) = \overline{F(a)}$ . An element  $a$  of  $\mathfrak{R}$  is *F-regular* if and only if  $a \in F(a)$ , and an ideal is *F-regular* if and only if each of its elements is *F-regular*. The *F-radical* of  $\mathfrak{R}$  is to consist of the elements of  $\mathfrak{R}$  which generate *F-regular* two-sided ideals. It is shown that  $\mathfrak{N}$  is a two-sided ideal in  $\mathfrak{R}$ , that  $\mathfrak{R}/\mathfrak{N}$  has zero *F-radical* and that, under certain conditions, the *F-radical* of the matrix ring  $\mathfrak{R}_n$  is  $\mathfrak{N}_n$ . The vanishing of the *F-radical* is necessary and sufficient that  $\mathfrak{R}$  be isomorphic to a subdirect sum of subdirectly irreducible rings of zero *F-radical*. Various choices of  $F$  are discussed, a case of special interest being  $F_1(a) = \{x + ax + \sum y_i z_i + \sum y_i a z_i; x, y_i, z_i \in \mathfrak{R}\}$ . In the presence of the descending chain condition for right ideals, the  $F_1$ -radical coincides with the radical as defined by Jacobson (Amer. J. Math. vol. 67 (1945) pp. 300–320) and also with the various other definitions. The vanishing of the  $F_1$ -radical is necessary and sufficient that  $\mathfrak{R}$  be isomorphic to a subdirect sum of simple rings, each with unit element. (Received January 30, 1946.)

#### 55. Marshall Hall: *The sum of continued fractions.*

By proving a general theorem on Cantor point sets, it is shown that every real number modulo 1 is the sum of two continued fractions whose partial quotients do not exceed 4. The number 4 is the best possible value for this theorem. As an appli-