COUNTABLE CONNECTED SPACES

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Introduction. Let $\mathfrak{S}$ be the class of all countable and connected perfectly separable Hausdorff spaces containing more than one point. It is known that an $\mathfrak{S}$-space cannot be regular or compact. Urysohn, using a complicated identification of points, has constructed the first example of an $\mathfrak{S}$-space.[1] Two $\mathfrak{S}$-spaces, X and X^*, more simply constructed and not involving identifications, are presented here. The space X^* is a connected subspace of X and contains a dispersion point; that is, the subspace formed from X^* by removing this one point is totally disconnected.

1. **Sequences.** The null sequence or any finite sequence of positive integers will hereafter be called more briefly a sequence. The null sequence or a sequence having an even number of elements is said to be even and a sequence having an odd number of elements is said to be odd. A sequence will usually be denoted by a lower case Greek letter: an arbitrary sequence by α, β, or γ; an arbitrary even sequence by λ, μ, or ν; the null sequence by o. A positive integer will be denoted by a lower case italic letter (not x, y, or z), which may also serve to represent the sequence consisting of that single integer.

The relation $\alpha \geqq i$ signifies that $a \geqq i$ for every element a of α, and $\alpha < i$ that $a < i$ for every element a of α. The null sequence vacuously satisfies both $o \geqq i$ and $o < i$.

The sequence formed by adjoining β to the end of α is denoted by $\alpha\beta$.

DEFINITION. *The relation $\beta \supset_i \alpha$ is to mean that a sequence α' exists such that $\beta = \alpha\alpha'$ and $\alpha' \geqq i$.*

Some immediate consequences of the preceding definitions are:

1.1. $\alpha \supset_i \alpha$.

1.2. *If $\beta \supset_j \alpha$ and $j \geqq i$, then $\beta \supset_i \alpha$.*

1.3. *If $\gamma \supset_i \beta$ and $\beta \supset_i \alpha$, then $\gamma \supset_i \alpha$.*

1.4. *If $\gamma \supset_a \alpha$ and $\gamma \supset_b \beta$, then $\beta \supset_a \alpha$ or $\alpha \supset_b \beta$.*

PROOF. Let $\gamma \supset_a \alpha$ and $\gamma \supset_b \beta$; then sequences α', β' exist such that

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[1] Paul Urysohn, *Über die Mächtigkeit der zusammenhängenden Mengen*, Math. Ann. vol. 94 (1925) pp. 262–295; see pp. 274–283 for the example.