7. Ernst Snapper: *Polynomial matrices in one variable, differential equations and module theory.*

This paper establishes the foundation for the theory of matrices $A=(\alpha_{ij})$, where $(\alpha_{ij})\in P[x_1, \cdots, x_n]$. Part I treats the case $n=1$. Contrary to the classical procedure which uses sub-determinants of A, the theory is developed intrinsically in terms of the column space C and row space R of A. The meanings of the irreducible factors and multiplicities of the norm and elementary divisor of A for C and R thus become clear. Systems of linear differential equations and algebraic equations are fully discussed. Part II reviews and extends the ideal theoretic module theory, developed by P. M. Grundy in *A generalization of additive ideal theory*, Proc. Cambridge Philos. Soc. vol. 38 (1942), and by the author in *Structure of linear sets*, Trans. Amer. Math. Soc. vol. 52 (1942). This theory is the foundation for the case $n>1$. A general theory of systems of linear equations over any ring $\mathfrak{r}$ is developed. All known criteria for the solvability of such systems for special rings are corollaries of the *criterion of lengths* of this general theory. If $\mathfrak{r}=P[x]$, the theory becomes the theory of Part I. (Received October 7, 1945.)

8. Ernst Snapper: *Polynomial matrices in several variables.*

This paper discusses the theory of matrices $A=(\alpha_{ij})$, where $\alpha_{ij}\in P[x_1, \cdots, x_n]$. The module theory, discussed in Part II of the author's paper *Polynomial matrices in one variable, differential equations and module theory*, associates several invariants to the column space C and the row space R of A, for example the associated primes $\mathfrak{p}_j$, the $\mathfrak{p}_j$-lengths, the $\mathfrak{p}_j$-elementary divisors, and so on. Since R and C are *polynomial* modules, the theory of the Hilbert characteristic function can be developed for them which gives rise to one further invariant, called the $\mathfrak{p}_j$-degree. In terms of these invariants, the theory of the system of linear partial differential equations and algebraic equations, represented by A, is investigated. Furthermore, the irreducible factors and multiplicities of the norm and elementary divisor of A, as defined by the author in *The resultant of a linear set*, Amer. J. Math. vol. 66 (1944), are explained in terms of the above invariants. (Received October 7, 1945.)

Analysis

9. N. R. Amundson: *On the boundary value problem of third kind for the quasi-linear parabolic differential equation.*

The author considers the quasi-linear parabolic equation with boundary conditions of the *third* kind for the open rectangle, that is, $u_{xx}=f(x, y, u, p, q)$; $-a_1u_x+b_1u=c_1(y)$, when $x=0$; $a_2u_x+b_2u=c_2(y)$, when $x=l$; $u=\phi(x)$, when $y=0$, where $c_i(y)$ and $\phi^{(iv)}(x)$ are continuous and b_i/a_i are non-negative constants. By use of the Green's function for the problem the above system is shown to be equivalent to a nonlinear integro-differential equation. Assuming that $f(x, y, u, p, q)$ is continuous in all five variables, and that its partial derivatives with respect to y, u, p, q satisfy a Lipschitz condition in u, p, q and are bounded, the *existence* of a solution $u(x, y)$ of the integro-differential equation is proved by an iteration method. Under the further assumption the u_x and u_y satisfy a Hölder condition with respect to y, the *uniqueness* of the solution $u(x, y)$ is established. M. Gevrey (*Thèse*, Journal de mathématique (6) vol. 9 (1913) and vol. 10 (1914)) considers the same differential equation for boundary conditions of the first kind. (Received October 19, 1945.)