A NOTE ON THE FIRST NORMAL SPACE OF A V_m IN AN R_n

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Let N be the normal plane at a point p of a surface V_2 in a Euclidean 4-space R_4. Calapso[2] proved that the hypersphere S in R_4 passing through p and with center c in N cuts V_2 in a curve with a double point at p, at which the two tangents to the curve coincide if and only if c lies on the Kommerell conic. The Kommerell conic is the locus of the point in which N (at p) is cut by the neighboring normal planes of V_2.

The purpose of this note is to generalize this result to the case of a subspace V_m in a Euclidean n-space R_n. To do this we shall first state some definitions and known results concerning the first (or principal) normal space of V_m in R_n.[3]

Let X^k $(k=1, \cdots, n)$ be the rectangular cartesian coordinates in R_n and let

$$X^k = x^k(u^a) \qquad (a, b, c = 1, \cdots, m) \tag{1}$$

be the equations of a V_m. Put

$$\overset{k}{B}_a = \partial_a \overset{k}{x} \equiv \partial x^k/\partial u^a. \tag{2}$$

Then the fundamental tensor and curvature tensor of V_m in R_n are, respectively,

$$'g_{cb} = \sum_k \overset{k}{B}_c \overset{k}{B}_b, \tag{3}$$

$$H_{cb}^{\cdot\cdot k} = \partial_c \overset{k}{B}_b - {'\Gamma}^a_{cb} \overset{k}{B}_a, \tag{4}$$

where $'\Gamma^a_{cb}$ is the Christoffel symbol of the second kind for V_m.

Let us consider the figure surrounding a certain point p of V_m. We have at p a tangent m-plane and a normal $(n-m)$-plane N. Let i^a be the unit tangent vector at p of an arbitrary curve in V_m passing through p. Then the component in N of the first curvature vector of the curve with respect to R_n is

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[2] R. Calapso, *Sulle reti di Voss di uno spazio lineare quadri dimensionale*, Rendiconti Seminario matematico Roma (4) vol. 2 (1938) pp. 276–311.

[3] See J. A. Schouten and D. J. Struik, *Einführung in der neueren Methoden der Differentialgeometrie* II, Groningen, 1938, chap. 3; D. Perepelkine, *Sur la courbure et les espaces normaux d'une V_m dans R_n*, Rec. Math. (Mat. Sbornik) N.S. vol. 42 (1935) pp. 81–100.