In a recent paper (A. N. Lowan and H. E. Salzer, *Table of coefficients for numerical integration without differences*, Journal of Mathematics and Physics vol. 24 (1945) pp. 1–21) quantities $B_i^{(n)}(p)$ are tabulated to 10 decimal places, for continuous numerical integration (that is, integration to various points within an interval of tabulation) by a Lagrangian formula which uses the tabular entries only. The present note indicates how those same quantities $B_i^{(n)}(p)$ can be employed as they stand, for continuous numerical integration using differences, in formulas obtained by integrating the interpolation formulas of Gregory-Newton, Newton-Gauss (two forms), Everett, and Steffensen. (Received August 6, 1945.)

Geometry

233. H. S. M. Coxeter: *Quaternions and reflections.*

Every quaternion $x=x_0+x_1i+x_2j+x_3k$ determines a point $P_x=(x_0, x_1, x_2, x_3)$ in Euclidean 4-space, and every quaternion a of unit norm determines a hyperplane $a_0x_0+a_1x_1+a_2x_2+a_3x_3=0$. The reflection in that hyperplane is found to be the transformation $x\rightarrow -a\bar{x}a$. This leads easily to the classical expression $x\rightarrow axb$ for the general displacement preserving the origin. If p and q are pure quaternions of unit norm, the transformation $x\rightarrow(\cos\alpha+p\sin\alpha)x(\cos\beta+q\sin\beta)$ represents the double rotation through angles $\alpha\pm\beta$ about the two completely orthogonal planes $P_0P_{p\mp q}P_{1\pm pq}$. (Received October 1, 1945.)

234. H. S. M. Coxeter: *The order of the symmetry group of the general regular hyper-solid.*

Schlafli defined $\{p, q, r\}$ as the regular four-dimensional polytope bounded by $\{p, q\}$'s, r at each edge; for example, $\{4, 3, 3\}$ is the hyper-cube. The order, g, of the symmetry group is found to be given by $16h/g=6/j_{p,q}+6/j_{q,r}+1/p+1/r-2$, where $\cos^2\pi/h$ is the greater root of the equation $x^2-(\cos^2\pi/p+\cos^2\pi/q+\cos^2\pi/r)x+\cos^2\pi/p\cos^2\pi/r=0$, and $j_{p,q}=[(2p+2q+7pq)/(2p+2q-pq)]^{1/2}+1$; for example, for the hyper-cube $\{4, 3, 3\}$, $128/g=6/8+6/6+1/4+1/3-2=1/3$. (Received October 1, 1945.)

235. H. S. M. Coxeter: *The Petrie polygon of a regular solid.*

Schlafli defined $\{p, q\}$ as the regular solid bounded by p-gons, q at each vertex; for example, $\{4, 3\}$ is the cube. The Petrie polygon of $\{p, q\}$ is a skew h-gon such that every two consecutive sides, but no three, belong to a face of the solid; for example, the Petrie polygon of the cube is a skew hexagon. It is found that $h=(g+1)^{1/2}-1$, where g is the order of the symmetry group (that is, four times the number of edges). Since $4/g=1/p+1/q-1/2$, we deduce an expression for h in terms of p and q. Moreover, the solid has $3h/2$ planes of symmetry. (Received October 1, 1945.)

236. M. M. Day: *Note on the billiard ball problem.*

In his book *Dynamical systems*, G. D. Birkhoff proves by the rather deep Poincaré ring theorem the fact that for each convex cornerless billiard table and each integer n there is a closed path around the table of precisely n sides such that a billiard ball will follow this path around and around if it satisfies the usual reflection law that the angle of incidence equals the angle of reflection whenever the ball hits a side. An elementary proof of this result is given by the following two statements: (1) Any n-sided convex polygon of stationary length inscribed in a convex cornerless curve satisfies the reflec-