characteristic roots of $A \bar{A}^{\prime}$, Browne has shown that $\rho_{1} \geqq|\lambda| \geqq \rho_{n}$. In this paper it is shown that for any complex number $\mu$ such that $\rho_{1} \geqq|\mu| \geqq \rho_{n}$ there exists a matrix $B$ which has the characteristic root $\mu$ and is such that $B \bar{B}^{\prime}=A \bar{A}^{\prime}$. Consideration is also given to the distribution of the roots of $B$ within the annular region. (Received August 10, 1945.)
207. Edward Rosenthall: On the sums of two squares and the sum of cubes.

In this paper the complete rational integer solution of each of the diophantine systems $x_{1}^{2}+y_{1}^{2}=x_{2}^{2}+y_{2}^{2}=\cdots=x_{n}^{2}+y_{n}^{2}$ and $\left(x_{1}^{2}+y_{1}^{2}\right)\left(x_{2}^{2}+y_{2}^{2}\right) \cdots\left(x_{n}^{2}+y_{n}^{2}\right)=\left(u_{1}^{2}+v_{1}^{2}\right)$ $\cdots\left(u_{m}^{2}+v_{m}^{2}\right)$ is obtained in terms of integral parameters. Also, a method is given for finding all the sets of rational integers satisfying the diophantine equation $x_{1}^{3}+x_{2}^{3}$ $+\cdots+x_{2 n}^{3}=0$. This method reduces the resolution of this equation to a system of linear homogeneous equations in which the number of unknowns always exceeds the number of equations. The above results are deduced from the complete integer solution in the quadratic fields $R a(i), R a\left((-3)^{1 / 2}\right)$ of certain connected simple and extended multiplicative equations. (Received September 4, 1945.)

## Analysis

## 208. R. P. Agnew: A simple sufficient condition that a method of summability be stronger than convergence.

Let $\sigma_{n}=\sum_{k=1}^{\infty} a_{n k} \rho_{k}$ be a regular Silverman-Toeplitz transformation by which a sequence $s_{1}, s_{2}, \cdots$ is summable to $\sigma$ if $\sigma_{n} \rightarrow \sigma$ as $n \rightarrow \infty$. It is shown that if $a_{n k} \rightarrow 0$ as $n, k \rightarrow \infty$, then some divergent sequences of zeros and ones are summable. A more general theorem applies to transformations which are not necessarily regular. If (i) $\sum\left|a_{n k}\right|<\infty$ for each fixed $n=1,2, \cdots$ and if (ii) as $n \rightarrow \infty$, the maximum for $k=1,2, \cdots$ of $\left|a_{n k}\right|$ converges to 0 , then some divergent series of zeros and ones are summable. The theorems furnish criteria for determination of relations among methods of summability. (Received August 20, 1945.)
209. R. P. Agnew: Characterization of methods of summability effective for power series inside circles of convergence.

A matrix $b_{n k}$ of real or complex constants determines a series-to-sequence transformation $\sigma_{n}=\sum_{k=0}^{\infty} b_{n k} u_{k}$ by means of which a series $\sum u_{n}$ is summable $B$ to $\sigma$ if $\sigma_{0}, \sigma_{1}, \cdots$ exist and $\lim \sigma_{n}=\sigma$. In order that the matrix $b_{n k}$ be such that $\sum u_{n}$ is summable $B$ whenever $\sum u_{n} z^{n}$ has radius of convergence greater than 1 , it is necessary and sufficient that (1) constants $\beta_{0}, \beta_{1}, \cdots$ exist such that $\lim _{n \rightarrow \infty} b_{n k}=\beta_{k}$ when $k=0,1,2, \cdots$ and (2) to each number $\theta$ in the interval $0<\theta<1$ corresponds a constand $M(\theta)$ such that $\left|b_{n k} \theta^{\theta}\right|<M(\theta)$ when $n, k=0,1,2, \cdots$. If (1) and (2) hold and $\sum u_{n} z^{n}$ has radius of convergence greater than 1 , then $\sum \beta_{n} u_{n}$ converges absolutely and the number $B\left\{\sum u_{n}\right\}$ to which $\sum u_{n}$ is summable $B$ is $\sum \beta_{n} u_{n}$. In order that $B\left\{\sum u_{n}\right\}=\sum u_{n}$ whenever $\sum u_{n}$ has radius of convergence greater than 1 , it is necessary and sufficient that (1) and (2) hold with $\beta_{k}=1$ for each $k$. (Received August 3, 1945.)

## 210. Joshua Barlaz: On some triangular summability methods.

A class of triangular sequence-to-sequence summability methods is given by the transform $t_{n}=e^{-x_{n}} \sum_{\nu=0}^{\infty} S_{\nu} x_{n}^{\nu} / \nu!, n=0,1,2, \cdots$, where $\left\{x_{n}\right\}$ is a sequence of numbers

