equations, $-\sum \nabla_{\nu}\left(|\psi|^{2} \nabla_{\nu} S\right)+(\partial / \partial t)|\psi|^{2}=0$, a continuity equation with density $|\psi|^{2}$ and streaming velocity $-\nabla_{\nu} S$, and $\left(\bar{\sigma}^{2} / 2|\psi|\right) \nabla^{2}|\psi|+\partial S / \partial t-c^{2}-U-\sum\left(\nabla_{\nu} S\right)^{2} / 2=0$, a Hamilton-Jacobi equation or Bernouilli equation with an assumed pressure function $\left(\bar{\sigma}^{2} / 2|\psi|\right) \nabla^{2}|\psi|$. An analysis of the motion of stars in a stellar system, treated as a hydrodynamical problem, shows that the above assumption about pressure function is plausible and fits the observed facts. Discrete effects (like quantum effects) cannot show up in this astronomical theory because observed hydrodynamical quantities in a stellar system are to be understood as averages taken over volumes containing many statistically independent elements (stars or star clusters), and they have to be confronted with a $\psi$ function which is a superposition of many stationary $\psi$. Because of its mathematical simplicity this theory provides for an approach to problems such as transient solutions of the wave equation, selfconsistent steady and transient fields (that is, Poisson equation and hydrodynamical equations combined). (Received August 1, 1945.)
171. Arturo Rosenbluth and Norbert Wiener: Mathematics of fibrillation and flutter in the heart.

The known facts about the continuation and refractory period of a muscle fiber are used to explain the phenomena of flutter and fibrillation in the vertebra heart. A geometrical discussion is given of the flutter problem while the fibrillation problem is reduced to a statistical form. (Received July 20, 1945.)

## 172. H. E. Salzer: Table of coefficients for repeated integration with differences.

For functions tabulated at a uniform interval, formulas for $k$-fold integration, using advancing or backward differences, are obtained by integrating the Gregory-Newton advancing-difference interpolation formula or the Newton backward-difference formula. The quantities $G_{n}^{(k)} \equiv(1 / n!) \times \int_{0}^{1} \cdots \int_{0}^{p} \int_{0}^{p} p(p-1) \cdots(p-n+1)(d p)^{k}$ and $H_{n}^{(k)} \equiv(1 / n!) \times \int_{0}^{1} \cdots \int_{0}^{p} \int_{0}^{p} p(p+1) \cdots(p+n-1)(d p)^{k}$ are the coefficients of the $n$th advancing and backward differences respectively in the formulas $\int_{x_{0}}^{x_{1}} \cdots \int_{x_{0}}^{x} \int_{x_{0}}^{x} f(x)(d x)^{k}$ $=h^{k}\left[f\left(x_{0}\right) / k!+\sum_{n=1}^{m} G_{n}^{(k)} \nabla^{n} f\left(x_{0}\right)\right]+R m=h^{k}\left[f\left(x_{0}\right) / k!+\sum_{n=1}^{m} H_{n}^{(k)} \nabla^{n} f\left(x_{0}\right)\right]+R_{m}^{\prime}$, where $x_{1}-x_{0}=h=$ the tabular interval. Previous tables (A. N. Lowan, H. E. Salzer, Journal of Mathematics and Physics vol. 22 (1943) pp. 49-50, and W. E. Milne, Amer. Math. Monthly vol. 40 (1933) pp. 322-327) furnish exact values for $k=1, n=1$ (1) 20 and $k=2, n=1$ (1) 7. The present table has exact values for $k=2, n=1$ (1) 20 and decimal values for $k=2$ (1) $6, n=1$ (1) $22-k$. The quantities $G_{n}^{(2)}$ and $H_{n}^{(2)}$ are expressed in several ways as functions of $B_{\nu}^{(n)}(x)$, Bernoulli polynomials of order $n$ and degree $\nu$, where $t_{n} e^{x t} /\left(e^{t}-1\right)^{n}=\sum_{\nu-0}^{\infty} t^{\nu} B_{\nu}^{(n)}(x) / \nu!$, and were checked in terms of previously tabulated values of $B_{\nu}^{(n)}(x)$. Also a simple recursion formula for $G_{n}^{(k)}$ in terms of $G_{n}^{(k-1)}$ and $G_{n+1}^{(k-1)}$ (and similarly for $H_{n}^{(k)}$ ), valid for $k \geqq 2$, was used for computation when $k>2$. (Received July 7, 1945.)

## Geometry

## 173. P. O. Bell: Power series expansions for the equations of a variety

 in hyperspace.For the study of the local properties of an arbitrary variety $V_{m}$ in a linear space $S_{n}$ a moving reference frame $F\left(x_{0}, x_{1}, \cdots, x_{n}\right)$ may be selected whose vertex $x_{0}$ is a generic point of $V_{m}$. The general projective homogeneous coordinates $x_{j}^{\boldsymbol{p}}$ of the points

