A CLASS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS AND THEIR PROPERTIES

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1. The class $\mathcal{C}$ of complex solutions of a linear partial differential equation. Any two real harmonic functions $U(x, y)$, $V(x, y)$—that is, solutions of the Laplace equation—can be combined to form a complex harmonic function $U+iV$. The class of *all* complex harmonic functions is of no interest, because in effect it possesses no special properties not already possessed by real harmonic functions. However, the theory of analytic functions of a complex variable, which is the subclass of complex harmonic functions where U and V satisfy the Cauchy-Riemann equations, has proved to be a powerful means for the study of *real* harmonic functions.

There is an analogous situation in the case of real solutions of the general linear equation of elliptic type,

$$U_{xx} + U_{yy} + A(x, y)U_x + B(x, y)U_y + C(x, y)U = 0,$$
$$U_x = \partial U/\partial x, \cdots .$$

As in the case of the Laplace equation, any two real solutions of this equation can be combined to form a complex solution, and it has been shown ([1a, 1b, 1c])[1] that there exists a certain subclass $\mathcal{C}$ of complex solutions which frequently aids in the study of real solutions in a manner which bears close analogies to the relationships between analytic functions and real harmonic functions in the case of the Laplace equation. Many properties of functions of class $\mathcal{C}$ are closely related to those of ordinary analytic functions: there exists a set of functions in class $\mathcal{C}$ which behave like powers of z; any function of class $\mathcal{C}$ can be expanded in a uniformly convergent series of these analogs of powers; any function of this class which is regular in a simply-connected domain can be approximated in this domain by the analog of a polynomial (see §4); singularities of functions of class $\mathcal{C}$ have properties analogous to those of analytic functions [1c, §§5–6]; and so on. Speaking generally, there exists a method of translating properties of analytic functions of a complex variable into properties of complex solutions of the elliptic type equation which belong to class $\mathcal{C}$ [1c, §1].

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[1] Numbers in brackets refer to the Bibliography at the end of the paper.