ON THE CAPACITY OF A CONDENSER

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1. Introduction. (a) Any student of electric phenomena is familiar with the *Leyden jar*. In its simplest form it consists of two circular cylinders, with a common axis, charged with electricity. If there is a difference in the potentials of the cylinders, then electric energy, which can be recovered at any time in the form of electric discharge, is stored up in the jar.

Let us denote the radii of the cylinders by a and b, $a<b$. Disregarding the effect of the ends, the potential u of the electric field between the cylinders depends only on the distance r from the axis, and so it must have the form

$$u = A \log r + B, \tag{1}$$

where A and B are constants. On the cylinders, which are equipotential surfaces, we have

$$u_a = A \log a + B, \qquad u_b = A \log b + B. \tag{2}$$

The density σ_a of the charge on the cylinder $r=a$ is

$$\sigma_a = -\frac{\epsilon}{4\pi}\left(\frac{\partial u}{\partial r}\right)_{r=a} = -\frac{\epsilon A}{4\pi a}, \tag{3}$$

where ϵ denotes the dielectric constant of the field. The density σ_b of the charge on the other cylinder will be

$$\sigma_b = \frac{\epsilon}{4\pi}\left(\frac{\partial u}{\partial r}\right)_{r=b} = \frac{\epsilon A}{4\pi b}\,. \tag{4}$$

Hence the charge on the cylinders, per unit length, will be $2\pi a\sigma_a = -\epsilon A/2$ and $2\pi b\sigma_b=\epsilon A/2$, respectively. On the other hand the potential difference is

$$u_b - u_a = A \log (b/a). \tag{5}$$

The ratio

$$\frac{\text{charge (per unit length)}}{\text{potential difference}} = \frac{\epsilon A/2}{A \log (b/a)} = \frac{\epsilon}{2 \log (b/a)} \tag{6}$$

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