ARITHMETIC UPON AN ALGEBRAIC SURFACE[1]

B. SEGRE

The title of my lecture is, I am afraid, probably misleading and certainly too ambitious. For, on the one hand, the connection between arithmetic and geometry suggested by it is not the modern development in divisors theory, but an application of algebraic geometry for arithmetical purposes. On the other hand, I shall confine the subject of this lecture to cubic surfaces in ordinary space, considered in the rational domain, so that a proper title would be for instance *The geometry of ternary cubic Diophantine equations.*[2] I prefer, however, the more ambitious and inaccurate one, as suggesting the possibility of similar investigations for other surfaces, possibly considered in more general arithmetical fields.

The short time at my disposal does not allow me to dwell on such extensions. I mention only that I have already completed an extensive arithmetical research on quartic surfaces; and that the whole subject—arithmetic upon an algebraic surface—seems to me to be so wide in scope, that I can envisage the possibility of further important developments.

Let us consider an ordinary space, where coordinates (x, y, z) are introduced and points at infinity are defined in the usual way. I call *rational* an algebraic surface, or curve, or point set of this space when it can be represented by one or more algebraic equations with rational coefficients,

$$F(x, y, z) = 0 \tag{1}$$

say. Moreover, I call *rational* any such equation, and also any polynomial such as $F(x, y, z)$.

The problem of finding the rational solutions in x, y, z of equation (1) can then be stated as that of determining the *rational points lying*

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[2] On this subject cf. B. Segre, *A note on arithmetical properties of cubic surfaces*, J. London Math. Soc. vol. 18 (1943) p. 24; *On a parametric solution of the equation $x^3+y^3+az^3=b$, and on ternary forms representing every rational number*, ibid. p. 31; *On ternary nonhomogeneous cubic equations with more than one rational solution*, ibid. p. 88; *A parametric solution of the indeterminate cubic equation $z^2=f(x, y)$*, ibid. p. 226; *A complete parametric solution of certain homogeneous Diophantine equations, of degree n in $n+1$ variables*, ibid. vol. 19 (1944) p. 46. Another paper of mine, *On arithmetical properties of singular cubic surfaces*, will appear shortly in the same journal.

The present lecture sums up rather sketchily my previous results, to which it adds a few more, as for example the geometric construction of p. 156. Full details, together with further results, will be found in a forthcoming extensive and systematic work.