where p, n, $d \equiv 0 \pmod 1$; $n < p$; $c = 0, 1, 2, \cdots, d-1$; and $m, q = 0, \pm 1, \pm 2, \cdots$.

Thus we have obtained transformations of the Mirimanoff and Vandiver congruences connected with the solution of equation (1.1). Other, and in some cases more symmetric, transformations of these congruences are possible by using one of the other permissible forms for the quadratic functional equation (2.11).

PURDUE UNIVERSITY

DEVELOPMENT OF CERTAIN QUADRATIC FUNCTIONAL EQUATIONS

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1. **Introduction.** In some work devoted to the derivation of certain congruences connected with the solution of Fermat's Last Theorem, it was found necessary to develop several quadratic functional equations of a particular function which we shall define later. This note will deal with the derivation of these functional equations. Maier[1] derived two such quadratic functional equations for a generating function of the Bernoulli polynomials. This work of Maier serves as the basis for our developments.

2. **The Maier results.** The function, $f(x, u)$, used by Maier was defined by the infinite series

$$\sum_{r=-\infty}^{+\infty} \frac{e^{2\pi i x r}}{u + r}, \tag{2.1}$$

where x is a real variable satisfying the inequality $0 < x < 1$, and where u is a real variable subject to the restriction $u \not\equiv 0 \pmod 1$. Maier, then, showed that if u, v, x, ξ are such that $u, v, (u+v) \not\equiv 0 \pmod 1$ and $0 < \xi < x < 1$, that the function $f(x, u)$ is a solution of the functional equation

$$\begin{aligned} f(x, u)f(\xi, v) = {} & f(\xi, u + v)f(x - \xi, u) \\ & - f(x, u + v)f(x - \xi, -v). \end{aligned} \tag{2.2}$$

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[1] W. Maier, *Zur Theorie der elliptischen Funktionen*, Math. Ann. vol. 104 (1930) pp. 745–769.