NOTE ON THE EXPANSION OF A POWER SERIES INTO A CONTINUED FRACTION

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1. **Introduction.** In view of the fact that the continued fraction frequently furnishes a method for summing a slowly convergent or even divergent power series, it is desirable to have a simple algorithem for obtaining the continued fraction. We present here such an algorithm based upon the fact that the process for constructing a sequence of orthogonal polynomials can be so arranged that it gives simultaneously a continued fraction expansion for a power series. It has been known at least since Tschebycheff that the problem of constructing a sequence of orthogonal polynomials is related to the problem of expanding a power series into a continued fraction. However, the fact that the two problems are actually identical does not seem to have been emphasized.

2. **The expansion of a power series into a *J*-fraction.** A continued fraction of the form

$$\frac{a_0}{b_1 + z} - \frac{a_1}{b_2 + z} - \frac{a_2}{b_3 + z} - \cdots \tag{2.1}$$

is called a *J-fraction*. The a_p and b_p are constants, and z is a complex variable. We shall suppose that the a_p are different from zero. We denote by $A_p(z)$ and $B_p(z)$ the pth numerator and denominator, respectively, of the J-fraction, so that $A_p(z)/B_p(z)$ is its pth approximant. The usual recurrence formulas

$$\begin{aligned} &A_0 = 0,\ A_1 = a_0, \qquad A_p = (b_p + z)A_{p-1} - a_{p-1}A_{p-2}, \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad p = 2, 3, 4, \cdots, \\ &B_0 = 1,\ B_1 = b_1 + z,\ B_p = (b_p + z)B_{p-1} - a_{p-1}B_{p-2}, \end{aligned} \tag{2.2}$$

show that $A_p(z)$ is a polynomial of degree $p-1$, and $B_p(z)$ is a polynomial of degree p:

$$\begin{aligned} A_p(z) &= \alpha_{p,0}z^{p-1} + \alpha_{p,1}z^{p-2} + \cdots + \alpha_{p,p-1}, \\ B_p(z) &= \beta_{p,0}z^p + \beta_{p,1}z^{p-1} + \cdots + \beta_{p,p}. \end{aligned} \tag{2.3}$$

We note that

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