THE SOLUTION OF THE DIFFERENTIAL EQUATION $(a^2\, \partial^2/\partial t^2 - \Delta)(\partial^2/\partial t^2 - \Delta)u = f(x, y, z, t)$ BY HADAMARD'S METHOD

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1. **Introduction.** Hadamard [1][1] generalized and extended Riemann's [2] method of solution of the initial value problem to any hyperbolic linear differential equation of the second order in any number of variables. The essential part of Hadamard's work is his discovery of the significance of an elementary solution which is a function of the geodetic distance determined by the characteristic manifold of the differential equation. Given the differential equation $L[u] = f$ and the initial data on a manifold C, let $\Gamma = 0$ be the equation of the characteristic manifold of the differential equation and $M[u]$ the adjoint expression of $L[u]$. The first step is the determination of an elementary solution V, that is, a solution of $M[V] = 0$ which is singular on $\Gamma = 0$. To obtain the solution of the initial value problem, Hadamard considers the domain bounded by the characteristic "conoid" through a point P, and the initial manifold $C = 0$. Since the elementary solution is singular on the characteristic conoid, in order to apply Green's formula Hadamard uses an interior domain $G_{\epsilon,\delta}$ which approaches the original domain as $\epsilon \to 0$ and $\delta \to 0$. For a fixed δ each term of Green's formula has then the following form:

$$B(\epsilon) = b + (1/\epsilon^{(n-2)/2})(b_0 + b_1\epsilon + \cdots + b_{(n-3)/2}\epsilon^{(n-3)/2}) + (\epsilon),$$

where (ϵ) tends to zero as $\epsilon \to 0$ and the b_i are independent of ϵ. The term b, the "finite part" of the integral, has the property to remain invariant under all transformations of the parameter ϵ. It follows from Green's formula that the sum of all such "finite parts" is equal to zero, which gives a relation for the function u. As we let the parameter δ approach zero we obtain the desired expression for $u(P)$. Hadamard, and independently Friedrichs [3], showed that in the case of an even number of variables each term of Green's formula will have an element containing $\log \epsilon$. The coefficients of $\log \epsilon$ have the property to remain invariant under all transformations of the parameter ϵ and it follows from Green's formula that the sum of all coefficients of $\log \epsilon$ is equal to zero, which gives the desired relation for the function u.

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[1] Numbers in brackets refer to the Bibliography at the end of the paper.