identity, the radical is defined by using the concept of a quasi-inverse due to Perlis. It is shown that N is a two-sided ideal and it coincides with the left radical, defined in a similar manner. If A has an identity, N is the intersection of the maximal right (left) ideals of A. The results of Stone and of McCoy are a consequence of this theorem. The author has also investigated the radical of an arbitrary algebra and of a normed ring and in the latter case has obtained the criterion: $z \in N$ if and only if $(za)^n \to 0$ for every a in A. (Received August 30, 1944.)

267. Jakob Levitzki: *Chain-conditions and nilpotency.*

Let S be an arbitrary ring. Denote by N_σ, N, and N_γ the sum of all nilpotent, semi-nilpotent and nil-ideals of S, respectively. R. Brauer (Bull. Amer. Math. Soc. vol. 48 (1942) pp. 752–758) by a simple argument proved that if the minimal condition holds for the ideals of S contained in N_σ, then N_σ is nilpotent. By a similar argument the author derived a characteristic condition for the nilpotency of N_σ (Duke Math. J. vol. 11 (1944) pp. 367–368). In the present note, characteristic minimal and maximal conditions are derived for the nilpotency of N_σ, N and N_γ. These results are corollaries of the following theorems: Denote by $R_1, R_2, \cdots$ resp. by $L_1, L_2, \cdots$ infinite sequences of right resp. left-ideals of a nil-subring T of S, then T is nilpotent if and only if: I. Each descending chain of the form $SL_1 \supset SL_1L_2 \supset \cdots$ and of the form $R_1S \supset R_2R_1S \supset \cdots$ is finite. II. Each ascending chain of the form $(O:L_1)_r \subset (O:L_1L_2)_r$ and of the form $(O:R_1)_l \subset (O:R_2R_1)_l \subset \cdots$ is finite. These theorems yield as a consequence various characteristic conditions for the semi-primarity of a ring. (Received September 7, 1944.)

Analysis

268. R. P. Agnew: *A simple and natural notation for the theory of summability of series and sequences.*

It is proposed that methods of summability be regarded as *operators*, and that the operational (that is, functional) notation be employed in the theory of summability. Thus the statement that a given sequence $s_0, s_1, s_2, \cdots$ or $\{s_n\}$ is summable to σ by a given method A is represented by $\sigma = A\{s_0, s_1, \cdots\}$ or $\sigma = A\{s_n\}$. The statement that a series $u_0 + u_1 + \cdots$ or $\sum u_n$ is summable B to σ is abbreviated to $\sigma = B\{u_0 + u_1 + \cdots\}$ or $\sigma = B\{\sum u_n\}$. Discussions and examples are given to illustrate the notation which, the author believes, should have been universally adopted many years ago. (Received September 28, 1944.)

269. R. P. Agnew: *Spans in Lebesgue and uniform spaces of translations of step functions.*

It is shown that, for each $p > 1$, the closure in the Lebesgue space L_p of the linear manifold determined by the translations of a given simple step function is the whole space L_p. An explicit formula is given for the approximation of one simple step function by linear combinations of translations of another. (Received September 23, 1944.)

270. E. F. Beckenbach: *Concerning the definition of harmonic functions.*

The following result, which may be compared with the Looman-Menchoff theorem concerning the Cauchy-Riemann first order partial differential equations, is established: If the real function $u(x, y)$ and its first order partial derivatives are continuous