A NOTE ON CHAIN CONDITIONS IN NILPOTENT RINGS AND GROUPS

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Maximal and minimal conditions for ideals in associative rings have often been considered, but little seems to be known of these conditions in non-associative rings, or of chain conditions on the non-normal subgroups of a group. Moreover, it is usual to assume the condition for one-sided ideals in noncommutative rings, and the weaker condition for two-sided ideals rarely appears. In this note we first consider a class of groups which are "nilpotent" with respect to a set of operators Ω. These groups include ordinary nilpotent groups, and associative and non-associative nilpotent rings and algebras as special cases. Our main theorem is to the effect that, for an Ω-nilpotent group, the maximal or minimal condition for Ω-subgroups implies the corresponding condition for all subgroups. As immediate consequences of this theorem it follows that, for nilpotent rings and algebras, the maximal or minimal condition for ideals implies the corresponding condition for modules, while for nilpotent groups, the maximal or minimal condition for normal subgroups implies the corresponding condition for all subgroups.

1. **Definition of Ω-nilpotency.** Let R be a group, and let Ω be a set of (left) operators of R which includes the inner automorphisms of R. R may also admit a second set of operators Φ, in which case it will be understood in what follows that all subgroups are supposed to admit Φ. The statement that a certain subgroup is an *Ω-subgroup* means that this subgroup admits Ω as well as Φ.

If A is any Ω-subgroup of R, A is normal, and we may suppose that R/A admits the same sets of operators, Ω and Φ, as R.

We take the following as our

DEFINITION OF AN Ω-NILPOTENT GROUP.[1] *R is Ω-nilpotent if there exists a strictly decreasing chain of Ω-subgroups*

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[1] Originally I discussed the chain theorems for groups for and rings separately, and while it was clear that the results were analogous, I did not succeed in unifying the proofs for the two cases. The referee, Dr. Irving Kaplansky, pointed out, however, that this could be accomplished by the use of the general notion of Ω-nilpotency which we give here, and which was suggested by him. I would like to express my thanks to the referee for his contribution to this paper.