THE COMPACTNESS OF THE RIEMANN MANIFOLD OF AN ABSTRACT FIELD OF ALGEBRAIC FUNCTIONS

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1. The existence of finite resolving systems. In an earlier paper[1] we have announced the result that the existence of a resolving system of the Riemann manifold of an abstract field of algebraic functions (in any number of variables) or—what is the same—the local uniformization theorem[2] implies the existence of *finite* resolving systems of the Riemann manifold. We have proved this result for algebraic surfaces by arithmetic considerations.[1] The proof for the general case of varieties, which at that time was in our possession,[3] and which we have promised to publish in a subsequent paper, was of similar nature, that is, it was based upon considerations involving the structure of certain infinite sequences of quotient rings. However, we have succeeded lately in finding a much simpler proof which is based on topological considerations.

Let Σ be a field of algebraic functions of several variables, over an arbitrary ground field k. By the Riemann manifold M of Σ we mean the totality of places of Σ, that is, the totality of zero-dimensional valuations v of Σ/k. If V is a projective model of Σ/k, and if H is any subset of V, we denote by $N(H)$ the subset of M consisting of those valuations $\mathfrak{v}$ which have center in H. By a resolving system of M we mean a collection $\mathfrak{V} = \{ V_a \}$ of projective models (finite or infinite in number) with the property that for any v in M there exists a V_a in $\mathfrak{V}$ such that the center of v on V_a is a simple point.

The topology which we introduce in M is simply this: *we choose as a basis for the closed sets of M the sets $N(W)$, where W is any algebraic subvariety of any projective model of Σ.* We prove that if topologized in this fashion, *the set M is a compact*[4] *topological space*. From this the result announced above follows immediately. For if $\{ V_a \}$ is a resolving system, and if we denote by S_a the singular locus of V_a, then $N(V_a - S_a)$ is an open set and $\{ N(V_a - S_a) \}$ is an open covering of M.

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[1] *A simplified proof for the resolution of singularities of an algebraic surface*, Ann. of Math. vol. 43 (1942) p. 583.

[2] See loc. cit. footnote 1.

[3] That proof was presented by us at a seminar in algebraic geometry at Johns Hopkins in 1942.

[4] We use the term compact in the same sense as it is used by S. Lefschetz in his *Algebraic topology* (Amer. Math. Soc. Colloquium Publications, vol. 27, 1942). The old term is bicompact.