BOOK REVIEW

Meromorphic functions and analytic curves. By Hermann Weyl and Joachim Weyl. (Annals of Mathematics Studies, no. 12.) Princeton University Press; London, Humphrey Milford and Oxford University Press, 1943. 9+269 pp. $3.50.

The theory of meromorphic functions began seriously with the now classical investigations of Poincaré, Hadamard, and Picard. After being continued by various other mathematicians it culminated in a paper by R. Nevanlinna in 1925, which was (according to the present author[1]) "one of the few great mathematical events in our century."

Nevanlinna associates with every meromorphic function $f(z)$ in the complex z-plane two non-negative real functions $N(r, a)$ and $m(r, a)$; the former counts how often f takes the value a in $|z| < r$, the latter measures the average closeness of f to a in $|z| < r$. Two of the principal results of Nevanlinna are these: (1) $N(r, a) + m(r, a) = T(r) + O(1)$, where $T(r)$ does not depend on a. (2) If any different complex numbers $a_1, \cdots, a_n$ and $\epsilon > 0$ are given, then for all $r > 0$ except for a set of finite measure, $\sum_i m(r, a_i) < (2+\epsilon)T(r)$. This implies in particular $N(r, a) \equiv 0$ for at most two a, that is, Picard's Theorem.

H. and J. Weyl and Ahlfors generalized these results in two respects. The underlying space is not any longer the z-plane, but any Riemann surface $\mathfrak{R}$, and the function f is replaced by $n+1$ functions on $\mathfrak{R}$ whose ratios define an analytic curve in the n-dimensional complex projective space P^n. Special cases are the algebraic curves (if $\mathfrak{R}$ is compact) and the meromorphic curves (if $\mathfrak{R}$ is the z-plane).

The first chapter provides the foundations for this theory and starts by introducing the Plücker coordinates of a $(p-1)$-dimensional linear subspace (p-spread) in P^n. The p-spreads appear as special vectors in the $C_{n,p}$-dimensional vector space of all p-ads. This space is metrized with the help of an Hermitian form in P^n. Next analytic curves are defined. An analytic curve C_1 determines for every p, $1 < p \leqq n$, the locus C_p of its osculating p-spreads in the space of all p-ads. The methods of Hensel-Landsberg allow to treat the stationary indices of these curves and their relations to each other. This investigation leads for algebraic curves to the general Plücker relations between the orders and the stationary indices of the C_p.

The analogue to Nevanlinna's result (1) is the subject of chapter II. The function $N(r, a)$ is replaced by a function $N_p(R, A)$, $1 \leqq p \leqq n$,

[1] The preface states that the book, while based on the joint work of the two authors, was actually written by H. Weyl.