THE STRUCTURE OF NORMED ABELIAN RINGS

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Introduction. A very prominent feature of the development of the theory of function spaces, a branch of mathematics which deservedly or not has attracted very concentrated attention since the turn of the century, is that one of the essential and defining features of a function was rapidly eliminated from attention. The property to which we refer is the ring property, more properly the multiplicative property, which demands that the multiplication of any two elements be allowed. Thus when the study of these spaces was sufficiently developed to be cast into abstract form, the basic domain of operations was not a ring but merely a group, an additive group or module, with real or complex operators. When a suitable topology is introduced into such a group it becomes a space. The spaces $\mathfrak{B}$ defined by Banach seem to merit the most attention. The topology introduced is of the simplest character: a metric, invariant under translation, and homogeneous with respect to scalar (that is, real or complex numerical) multiplication. The elements of these metric groups are called vectors.

It is only very recently that its birthright has been restored to this theory and that function spaces are being studied as rings and not merely as groups. The present address will attempt to delineate some aspects of the recent developments, to point to certain achievements, and to suggest some problems. The compilation of problems, the formulation of open questions, is usually a hazardous matter. It is frequently difficult to understand the importance of a question before it has been answered. Thus our suggestions will be largely tentative.

Definition and examples of normed rings. To begin with, a Banach space is an additive group of elements $a, b, c, f, g, \cdots$ with operators $\alpha, \beta, \lambda, \mu, \cdots$ which are complex numbers (in the present work real scalars will be excluded). Every element a has a norm $|a|$ (also written $\|a\|$ or even $|||a|||$!) which is a non-negative number. The space is metrized by the norm with dist $(a, b) = |b-a|$; it is complete in this norm. Finally the norm is homogeneous, that is, $|\alpha a| = |\alpha| \cdot |a|$. We are now in a position to define a normed abelian ring.

An address delivered before the New York meeting of the Society on April 29, 1944, by invitation of the Program Committee; received by the editors May 2, 1944.