ON RIESZ SUMMABILITY OF FOURIER SERIES BY EXPONENTIAL MEANS

FU TRAING WANG

Let $f(t)$ be an integrable periodic function with the period 2π. Let its Fourier series be

$$f(t) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt \tag{1}$$

and let

$$\phi(t) = \{f(x+t) + f(x-t) - 2s\}/2,$$

$$\phi_\beta(t) = (1/\Gamma(\beta)) \int_0^t (t-u)^{\beta-1}\phi(u)du,$$

$$A_n = a_n \cos nx + b_n \sin nx.$$

We shall prove the following result.[1]

If $A_n > -Kn^{-\beta/\gamma}$ $(\gamma > \beta > 0)$ *and*

$$\phi_\beta(t) = o(t^\gamma) \qquad \textit{as } t \to 0, \tag{2}$$

the Fourier series (1) *converges to* s *at* $t = x$.

Set $\alpha = 1 - \beta/\gamma$, and

$$C_\tau(\omega) = a_0 e^{\tau\omega^\alpha}/2 + \sum_{n<\omega} (e^{\omega^\alpha} - e^{n^\alpha})^\tau A_n. \tag{3}$$

The Fourier series (1) is said to be summable (e^{n^α}, τ) to the sum s if[2]

$$C_\tau(\omega) = se^{\tau\omega^\alpha} + o(e^{\tau\omega^\alpha}) \qquad \text{as } \omega \to \infty.$$

Concerning this kind of summability we have the following theorem.

THEOREM.[3] *If* (2) *holds and* τ *is a positive integer greater than* $\gamma + 1$ *the Fourier series* (1) *is summable* (e^{n^α}, τ) *to the sum* s *at* $t = x$.

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[1] G. H. Hardy and J. E. Littlewood [2], F. T. Wang [6]. Numbers in brackets refer to the references listed at the end of the paper.

[2] G. H. Hardy and M. Riesz [3].

[3] Under the hypotheses of the Theorem I have established that the Fourier series (1) is summable $(e^{n^\alpha}, \gamma+\delta)(\delta > 0)$ to the sum s at $t = x$, but the proof is very complicated. See F. T. Wang [6].