# EQUICONVERGENCE THEOREMS FOR ORTHONORMAL POLYNOMIALS

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**1. Introduction.** If $\rho(x)$ is a non-negative function integrable on the finite interval $(a, b)$ and positive on a set of positive measure, there exists a unique set of polynomials $\{p_n(\rho, x)\}$ of degree $n$, $p_n(\rho, x) = c_n x^n + \cdots$, $c_n > 0$, which are orthonormal on $(a, b)$ relative to the weight function $\rho(x)$; that is

$$\int_a^b \rho(x) p_n(\rho, x) p_m(\rho, x) dx = \begin{cases} 0 & \text{if} \quad n \neq m, \\ 1 & \text{if} \quad m = n. \end{cases}$$

In the sequel the abbreviation ONP will be used to denote such a set of polynomials.

Given any function $f(x)$ for which the integral $\int_a^b \rho(x) f(x) dx$ exists, the set of ONP associated with $\rho(x)$ may be used to construct the formal expansion

$$(1) \qquad f(x) \sim \sum_{n=0}^{\infty} a_n p_n(\rho, x), \qquad a_n = \int_a^b \rho(t) f(t) p_n(\rho, t) dt.$$

Much attention has been given to the convergence properties of such series for particular choices of $\rho(x)$. In the succeeding pages known results on this problem are extended by means of what seems to be a new type of proof.

Let $\{p_n(\rho_1, x)\}$ and $\{p_n(\rho_2, x)\}$ be two sets of ONP with different weight functions $\rho_1(x)$ and $\rho_2(x)$ and let

$$(2) \qquad s_n(f; \rho_i, x) = \sum_{k=0}^{n} a_{ki} p_k(\rho_i, x), \qquad i = 1, 2,$$

denote the partial sums of $n$th degree for the two corresponding expansions (1) associated with $f(x)$. In §§5 and 6 below certain sufficient conditions will be established for the validity of the relation

$$(3) \qquad \lim_{n \to \infty} \{ s_n(f; \rho_1, x) - s_n(f; \rho_2, x) \} = 0;$$

that is, conditions under which the two series mentioned are *equiconvergent*. It should be stated that the emphasis of the paper is upon method rather than upon specific conditions. The discerning reader

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