## ABSTRACTS OF PAPERS

## SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

## Algebra and Theory of Numbers

103. Leon Alaoglu and Paul Erdös: On highly composite and similar numbers.

A number *n* is highly composite if for all m < n, d(m) < d(n); superabundant if for all m < n,  $\sigma(m)/m < \sigma(n)/n$ ; and highly abundant if for all m < n,  $\sigma(m) < \sigma(n)$ . If  $q^k$  is the highest power of the prime *q* dividing the highly abundant number *n*, it is found that with at most  $c(\epsilon) \log_2 n / \log_3 n$  exceptions,  $(1-\epsilon) \log n \log_2 n / q < q^k \log q < (1+\epsilon) \log n \log_2 n$ . The superabundants satisfy a stronger inequality with no exceptions, and a similar formula is proved for the highly composite numbers. For these two latter classes the inequalities determine the prime power factors almost uniquely. Highly composite numbers were introduced by Ramanujan. Pillai's D.Sc. thesis may contain related results, but it was never published and is inaccessible. (Received February 14, 1944.)

## 104. S. P. Avann: Relations between join-irreducibles and meet-irreducibles in a modular lattice. I.

Let  $\tau$  and  $\tau'$  be the respective orders of the partially ordered sets P of joinirreducible elements and P' of meet-irreducible elements in a finite modular lattice L. It is proved that  $\tau = \tau'$ , if certain restrictions are made upon the quotient sublattice structure of L. A reasonable conjecture is that  $\tau = \tau'$  without restriction, since P and P' are even isomorphic when L is distributive. In a complete set  $Q_i$  of projective prime quotients of L the number of minimal quotients, characterized by join-irreducible numerators, is equal to the number of maximal quotients, characterized by meetirreducible denominators, if and only if  $\tau_i = \tau'_i$  for the simple homomorphic image  $L_i$  of L corresponding to  $Q_i$ . (Received March 29, 1944.)

105. Reinhold Baer: Groups without proper isomorphic quotient groups.

This paper contains a discussion of groups G with the following property: If N is a normal subgroup of G, and G and G/N are isomorphic groups, then N=1. (Received February 2, 1944.)

106. Garrett Birkhoff: Subdirect products in universal algebra.

It is proved that any abstract algebra A (in a very general sense) is a subdirect union of subdirectly irreducible algebras. This theorem contains as special cases