BOOK REVIEW

Metric methods in Finsler spaces and in the foundations of geometry. By Herbert Busemann. Princeton University Press, 1942. 2+243 pp., 22 fig. $3.00.

Metric notions have entered the foundations of geometry in various ways. In his *Grundlagen*, Hilbert formulates axioms of congruence which imply the availability of a metric, but the idea of distance is not explicitly used. In the projective framework for the classical noneuclidean geometries (for example, Klein), distance is defined in terms of specific configurations which are themselves the product of rather elaborate preparatory study. In the differential approach (for example, Cartan), the theory of coördinate manifolds is taken for granted, distance is first introduced locally, and even after its extension to nonlocal measurements it plays little essential part in the investigations. Others (Lie, Hilbert (Anhang IV to the *Grundlagen*)) based their work on a group of transformations (motions) which provided the means for defining congruence. The advent of metric spaces (Fréchet) raised the possibility of beginning with metric ideas and developing from them the equipment and results of the earlier theories. Menger isolated and studied the properties a metric space must have in order to possess geodesics which behave conveniently. The present monograph takes up at that point and develops a connected theory, occasional portions of which have appeared elsewhere, of metric spaces with geodesics.

The object of study is a finitely compact (hence separable) metric space Σ which is (internally) convex in the sense that, if $X\neq Z$, there is a point Y between X and Z (that is, such that $XY+YZ=XZ$). Define a segment to be a congruent map of a closed real interval, and a geodesic to be a locally congruent map of the real axis. As a decisive local restriction (D), each point of Σ is assumed to have a neighborhood N such that, if A and B are any distinct points of N, the set of points between A and B forms a segment which can be extended appreciably (in fact, outside N), in either direction past A and B, and which is the ***unique*** segment between any pair of its points of which neither is too far beyond A or B. Without Axiom D, any two points P and Q can be joined by a segment. With Axiom D, this segment turns out to be unique, if P and Q are not too far apart; further, any pair can be connected by a geodesic, and a given segment can be embedded in one and only one geodesic. If there is a point at which