As for the isomorphism, we need only consider the case where V is defined from G. Obviously, the correspondence $ab^{-1} \rightleftarrows ab$ is one-one from G onto V. Moreover, the correspondent of $ab+cd$ is ah^{-1} where $dc^{-1}=hb^{-1}$ or $h^{-1}=b^{-1}cd^{-1}$. This establishes the isomorphism.

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AN INVARIANT OF INTERSECTION OF TWO SURFACES

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1. **Introduction.** Projective invariants of several pairs of surfaces have been deduced and characterized geometrically by various authors.[1] In this paper we shall supplement their investigations by studying in ordinary space two surfaces intersecting at an ordinary point with distinct tangent planes.

In §2 we show by analysis the existence of a projective invariant determined by the neighborhood of the second order of the two surfaces at the point of intersection.

The final two sections are devoted to the presentation of projectively, as well as metrically, geometric characterizations of this invariant.

2. **Derivation.** Suppose that two surfaces S_1, S_2 in ordinary space intersect at an ordinary point O with distinct tangent planes τ_1, τ_2, and let the common tangent t be distinct from the asymptotic tangents. Let t_1, t_2 be the harmonic conjugate lines of t with respect to the asymptotic tangents of the surfaces S_1, S_2, respectively, at the point O. If we choose the point O to be the origin, the lines t, t_2, t_1 to be, respectively, the axes x, y, z of a general nonhomogeneous projective coordinate system, then the power series expansions of the surfaces S_1, S_2 in the neighborhood of the point O may be written in the form

$$S_1:\quad y = l_1x^2 + m_1z^2 + \cdots, \tag{1}$$

$$S_2:\quad z = l_2x^2 + m_2y^2 + \cdots. \tag{2}$$

Presented to the Society, September 13, 1943; received by the editors June 15, 1943.

[1] See the bibliography at the end of the paper.