

tive, real-valued, non-negative, finite measure function on E , vanishing only for ϕ , the zero of E . E is given the star-topology of G. Birkhoff, and it is shown that E will have such a measure if and only if it is metrizable in a certain way in this topology. It is next shown that E is a topological space if and only if it satisfies a certain distributive law. A basis for the neighborhoods of ϕ can then be characterized algebraically, making it possible to state simple algebraic equivalents for the various separation axioms. If E satisfies the countable chain condition, T_3 implies metrizability, which gives an outer measure on E . A measure is obtained by means of an additional algebraic requirement. Thus, if E satisfies the distributive law referred to, the countable chain condition, the algebraic equivalent of T_3 , and the additional requirement, there exists a measure-function on E . These conditions are easily seen to be necessary. It is not known whether they are independent. (Received October 2, 1943.)

250. R. M. Thrall: *On the decomposition of modular tensors. II.*

Let G be the n -rowed full linear group over a field k of characteristic p . A representation of G is called a tensor representation if its space is a direct sum of subspaces and factor spaces of tensor spaces. A main result of the present paper is that for a finite field k , the k -group ring of G has a faithful tensor representation. In paper I the representations afforded by all tensors of rank $m < 2p$ were determined subject to the condition that k has more than p elements. In this paper the same is done for the field k with p elements. A main tool in this investigation is the construction of a representation of G from each irreducible representation of the non-modular full linear group, and a corresponding extension of the Brauer-Nesbitt modular character theory to this case. The presence of zero divisors in the ring of polynomial functions over a finite field enters into the treatment of the case of tensors of rank $2p-1$ over a two-dimensional vector space, and the situation in that case should help point the way to the general decomposition theory. (Received October 1, 1943.)

ANALYSIS

251. Stefan Bergman: *Fundamental solutions of partial differential equations of the second order.*

As was previously shown, for every differential equation $L(U) = U_{z\bar{z}} + H(Z, \bar{Z})U = 0$, $Z = X + iY$, $\bar{Z} = X - iY$, there exists a function $E(Z, \bar{Z}, t) = 1 + Z\bar{Z}t^2 E^*(Z, \bar{Z}, t)$ such that $U = P(f) \equiv \int_{-1}^{+1} E(Z, \bar{Z}, t) f(Z(1-t^2)/2) dt / (1-t^2)^{1/2}$, where f is an arbitrary analytic function, is a solution of $L(U) = 0$ (see Duke Math. J. vol. 6 (1940) p. 537). The author shows that a fundamental solution $\Gamma(z, \bar{z}, \xi, \bar{\xi})$ of the equation $S(v) = v_{z\bar{z}} + F(z, \bar{z})v = 0$ is given by $P(1/2\pi) \log |Z| + G(Z, \bar{Z})$. Here $Z = z - \xi$, $\bar{Z} = \bar{z} - \bar{\xi}$, and P is the operator introduced above for the equation $L(U) = 0$ with $H(Z, \bar{Z}) = F(Z + \xi, \bar{Z} + \bar{\xi})$. $G(Z, \bar{Z}) = -\int_0^1 \int_0^1 D(Z, \bar{Z}) dZ d\bar{Z} + \int_0^1 \int_0^1 H(Z, \bar{Z}) (\int_0^1 \int_0^1 D(Z, \bar{Z}) dZ d\bar{Z}) dZ d\bar{Z} + \dots$ where $D(Z, \bar{Z}) = (1/2) \int_{-1}^{+1} t^2 [2E^* + \bar{Z}E_{\bar{z}}^* + ZE_z^*] dt / (1-t^2)^{1/2}$. Using the representations of functions $v(z, \bar{z})$, $S(v) = 0$, in the form of a line integral over a closed curve in terms of Γ , $\partial\Gamma/\partial n$, v and $\partial v/\partial n$ the author studies the growth of v and $\partial v/\partial n$ along circles $|z| = r$, $r \rightarrow \infty$. The existence of an analogous function $E(X, Y, t)$ for every equation $H(U) = U_{XY} + H(X, Y)U = 0$ (of hyperbolic type) has been established (see above reference). $(1/2\pi) \int_{-1}^{+1} E(X, Y, t) dt / (1-t^2)^{1/2}$ is now shown to be the Riemann function of the equation $v_{xy} + F(x, y)v = 0$, where $X = x - \xi$, $Y = y - \eta$ and $H(X, Y) = F(X + \xi, Y + \eta)$. Analogous relations hold for more general equations. (Received September 11, 1943.)