depend on the remainder in the division of $t^{k}$ by $\psi_{m}(t)-F_{m}$ and $\psi_{m}(t)$ respectively. (Cf. H. L. Lee, Duke Math. J. vol. 9 (1943) pp. 277-292.) (Received July 19, 1943.)

## 202. W. V. Parker: Limits to the characteristic roots of a matrix.

Let $A=\left(a_{i j}\right)$ be a square matrix of order $n$ with elements in the field of complex numbers; and define $S_{i}=\sum_{j=1}^{n}\left|a_{i j}\right|, T_{j}=\sum_{i=1}^{n}\left|a_{i j}\right|, U_{i}=2\left|a_{i i}\right|-S_{i}$, and $V_{j}=2\left|a_{j i}\right|$ $-T_{j}$. Let $S, T$ be the greatest of the $S_{i}, T_{j}$, respectively; and let $U, V$ be the least of the $U_{i}, V_{j}$, respectively. It is shown that the absolute value of each characteristic root of $A$ is not less than the greater of the numbers $U$ and $V$ and is not greater than the smaller of the numbers $S$ and $T$. Similar bounds are also found for the real and imaginary parts of the characteristic roots. (Received July 23, 1943.)

## 203. H. E. Salzer: Table of first two hundred squares expressed as a sum of four tetrahedral numbers.

The following empirical theorem is conjectured: Every square integer is expressible as the sum of four positive (including zero) tetrahedral numbers $\left(n^{3}-n\right) / 6$. It has been verified by a table prepared for the first 200 squares. This empirical theorem is a partial improvement of the statement that five non-negative tetrahedrals suffice for any integer. (See F. Pollock, Proc. Roy. Soc. London Ser. A. vol. 5 (1850).) (Received June 4, 1943.)

## Analysis

## 204. R. H. Cameron and W. T. Martin: Transformations of Wiener integrals under translations.

Let $F[y]$ be a functional defined and Wiener summable over the space $C$ consisting of all functions $x(t)$ continuous in $0 \leqq t \leqq 1$ and vanishing at $t=0$. In addition, let $F$ be continuous and let it be bounded over every bounded set $x(\cdot)$ of $C$. ( $F$ is called continuous if $F\left[y^{(n)}\right] \rightarrow F\left[y^{(0)}\right]$ whenever $y^{(n)}(t) \rightarrow y^{(0)}(t)$ uniformly in $0 \leqq t \leqq 1$, and $F$ is bounded over every bounded set $x(\cdot)$ of $C$ if for every positive constant $B$ there exists a constant $K=K_{B}$ such that $|F[y]| \leqq K$ for all $y(\cdot)$ of $C$ for which $|y(t)| \leqq B, 0 \leqq t \leqq 1$.) Under these conditions on the functional $F$ the authors obtain a transformation formula for Wiener integrals under translations of the form $y(t)=x(t)+x_{0}(t)$ where $x_{0}(t)$ is a given function of $C$ with a first derivative $x_{0}^{\prime}(t)$ of bounded variation in $0 \leqq t \leqq 1$. The transformation formula is $\int_{C}^{w} F[y] d_{w} y=\int_{C}^{w} F\left[x+x_{0}\right]$ $\exp \left\{-\int_{0}^{1}\left[x_{0}^{\prime}(t)\right]^{2} d t-2 \int_{0}^{1} x_{0}^{\prime}(t) d x(t)\right\} d_{w} x$. The formula forms a basis for the calculation of various types of Wiener integrals. (Received July 30, 1943.)

## 205. M. M. Day: Uniform convexity. IV.

In this paper relationships between uniform convexity, factor spaces, and conjugate spaces are discussed. Theorem 1: A normed vector space $B$ is uniformly convex if and only if all the two dimensional factor spaces of $B$ are uniformly convex with a common modulus of convexity. The concept of uniform flattening is suggested by a description of a "sharp edge" on the unit sphere in terms of the norm of the space. It is shown [Theorem 2] that this is dual to uniform convexity; that is, $B\left[B^{*}\right]$ is uniformly flattened if and only if $B^{*}[B]$ is uniformly convex. It follows that a complete uniformly flattened $B$ is reflexive. The proof of Theorem 2 uses a computation for

