definitions, then the loop integral $I(z, \beta)$ in (10) is developable asymptotically in the form

$$I(z, \beta) \sim \sum_{n=0}^{\infty} \frac{c_n}{[\log(-[\pm z])]^{\beta+n}\Gamma(1-\beta-n)}.$$

It thus appears that the presence of an algebraic singularity of $g(w)$ presents no serious difficulty.

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ON SOME FORMULAS INVOLVING THE DIVISOR FUNCTION

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Viggo Brun[1] has proved the formulas

$$T_1(n) - T_2(n) + T_3(n) - \cdots = -\mu(n), \qquad n > 1, \tag{1}$$

$$h(n) = T_1(n) - (1/2)T_2(n) + (1/3)T_3(n) - \cdots = \begin{cases} 0 \text{ if } n \text{ is not a prime power,} \\ 1/t \text{ if } n = p^t,\ p \text{ a prime;} \end{cases} \tag{2}$$

where $T_l(n)$ is the number of ways that n can be expressed as a product of l factors, each greater than 1. He obtains them as special cases of combinatorial theorems. Pavel Kuhn[2] has also given proofs but it seems that no one has attempted to give elementary number theory proofs of these formulas. It is the purpose of this note to give such proofs and to point out a few other formulas similar to (1) and (2).

All the formulas which we shall prove can be proved very concisely by using the generating function

$$\sum_{n=1}^{\infty} T_l(n)n^{-s} = \{\zeta(s) - 1\}^l,$$

and some simple properties of the zeta-function.[3] Our number theory

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[1] Netto, *Lehrbuch der Combinatorik*, 2d edition, 1927, chap. 14, especially pp. 276–277.

[2] *Det Kongelige Norske Videnskabers Selskab*, Forhandlinger, 1939.

[3] Interchanging the order of summation we have $\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{l-1}T_l(n)n^{-s} = \sum_{l=1}^{\infty}(-1)^{l-1}\{\zeta(s)-1\}^l = -\zeta(s)^{-1} = -\sum_{n=1}^{\infty}\mu(n)n^{-s}$, and (1) is obtained by comparing coefficients of n^{-s} in the two members. Similarly, (2) follows from $\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}(-1)^{l-1}l^{-1}T_l(n)n^{-s} = \sum_{l=1}^{\infty}(-1)^{l-1}l^{-1}\{\zeta(s)-1\}^l = \log\zeta(s) = \sum_p \log(1-p^{-s})^{-1} = \sum_p\sum_{t=1}^{\infty}t^{-1}p^{-ts}$.