TRANSFORMATIONS OF MULTIPLE FOURIER SERIES

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1. **Introduction.** The object of the present paper is the study and characterization of certain classes of factor sequence transformations of multiple Fourier series. A recent moment problem solution[1] by the author and a scheme of summation of multiple Fourier series developed by Bochner[2] are used in the study. The results include and extend known results for single Fourier series.

2. **Definitions and notation.** Let n be a positive integer, fixed but arbitrary. R^n will denote the euclidean n-space. (x), (y), and so on will denote $(x_1, x_2, \cdots, x_n)$, $(y_1, y_2, \cdots, y_n)$, and so on, points of R^n. ν, τ, j, k, s will be used for non-negative integers, and (ν), (τ), and so on will be used for $(\nu_1, \nu_2, \cdots, \nu_n)$, $(\tau_1, \tau_2, \cdots, \tau_n)$, lattice points of R^n. (0) will mean $(0, 0, \cdots, 0)$, and $(x)=(y)$ will mean $x_j=y_j$, $j=1, 2, \cdots, n$. $(k\cdot x)$ will stand for the number $k_1x_1+k_2x_2+ \cdots +k_nx_n$, $|x|$ for the number $(x_1^2+x_2^2+ \cdots +x_n^2)^{1/2}$. A, I, and λ will be used for functions defined on the lattice points of R^n, and I will be the characteristic function of the lattice points of R^n. E will be the set $E_{(x)}(-\pi \leqq x_j < \pi, j=1, 2, \cdots, n)$. R and t will be used for real numbers. $(x+y)$ will stand for $(x_1+y_1, x_2+y_2, \cdots, x_n+y_n)$, and $B(n)$ for a real constant depending only on n. Φ will be used for a function U^* of bounded variation in the sense of Saks, and if $\Phi(H) = \Phi_1(H)+\Phi_2(H)$ for any Borel set H with $\Phi_1(H) \geqq 0 \geqq \Phi_2(H)$ we will write $\int_H f(x)|d\Phi(E)|$ for $\int_H f(x)d\Phi_1(E) - \int_H f(x)d\Phi_2(E)$. When Φ is the Lebesgue measure function we will write $\int_H f(x)dx$ for

$$\int_H f(x)d\Phi(E).$$

We will write $f \in L$ to indicate that $\int_E f(x)dx$ exists, and $f \in C$ to indicate that f is continuous on $\overline{E}$ and $f(x)=f(x+y)$ for all combinations of $y_j=0$ or 2π, $j=1, 2, \cdots, n$. A function f defined over E will be defined over R_n by the extension $f(x)=f(x+y)$ with $y_j=0$ or 2π, $j = 1, 2, \cdots, n$.

Let

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[1] L. B. Hedge, *Moment problem for a bounded region*, Bull. Amer. Math. Soc. vol. 47 (1941) pp. 282–285. Referred to later as MP.

[2] S. Bochner, *Summation of multiple Fourier series by spherical means*, Trans. Amer. Math. Soc. vol. 40 (1936) pp. 175–207.