FUNCTIONAL TOPOLOGY

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Introduction. The present paper gives the fundamental existence proof for a homotopic critical point under hypotheses which are less stringent than those employed in the author's recent fascicule on functional topology and abstract variational theory [2]. This relaxation of hypotheses seems to be necessary if one is to extend the variational theory in the large to non-regular problems. Non-regular integrals include some of the most important and interesting integrals such as the Jacobi least action integral[1] in the three body problem of celestial mechanics. The author believes that a topological basis for the planetary orbits will be disclosed by studying the contour manifolds of this Jacobi integral.

The results of this paper will be used to extend the theory in the large to non-regular problems in two papers by Morse and Ewing.

1. **The metric spaces** M, L, J. We are concerned with a compact metric space M of elements $p, q, r, \cdots$ and distance $pq, pr, \cdots$. We shall deal with two functions $J(p)$ and $L(p)$, bounded, single-valued, and lower semi-continuous on M. In the applications p will be a curve joining two fixed points in some space, the distance pq will be the Fréchet distance between curves, while $J(p)$ and $L(p)$ will be integrals along p, with L the length of p.

Beside the metric M we shall use two other metrics, an L-metric with a distance

$$|pq| = pq + |L(p) - L(q)|,$$

and a J-metric with a distance,

$$pq + |J(p) - J(q)|.$$

We shall refer to the corresponding spaces as the spaces L and J. We shall make the following hypothesis.

HYPOTHESIS. *Convergence on L to a point p shall imply convergence on J to p.*

We shall not assume that convergence on J to p implies convergence on L to p. Convergence on L or J clearly implies convergence on M. Terms such as neighborhood, compact, and so on

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[1] For other examples see paper [1] by McShane.