#### 86. W. K. Feller: On a probability limit theorem of H. Cramér.

Let  $\{X_k\}$  be mutually independent random variables whose first moments vanish and whose second moments are  $\sigma_k^2$ ; let  $s_n^2 = \sigma_1^2 + \cdots + \sigma_n^2$ . In various applications one is concerned with the distribution function  $Pr\{X_1 + \cdots + X_n > xs_n\}$ , where  $x \to \infty$  as  $n \to \infty$ . The simplest binomial case has been studied by A. Khintchine, P. Lévy, Smirnoff and others. H. Cramér found a complete description of the asymptotic behavior of the above sums in the case where all  $X_k$  have the same distribution function. This theorem is generalized to the case of unequal components. The theorem is to serve as a base for a solution of the "problem of the iterated logarithm" in the general case. (Received November 21, 1942.)

# 87. W. K. Feller: On the general form of the so-called law of the iterated logarithm.

Let  $\{X_k\}$  be a sequence of mutually independent random variables whose first moments vanish and whose second moments are  $\sigma_k^2$ ; let  $s_n^2 = \sigma_1^2 + \cdots + \sigma_n^2$ . A sequence of numbers  $\phi_k \uparrow \infty$  is said to be of upper (lower) class if the probability that  $X_1 + \cdots + X_n > s_{n_k} \phi_{n_k}$  for infinitely many k is one (zero); any sequence  $\{\phi_k\}$  is either of upper or of lower class. A n.a.s. condition is found for a sequence  $\{\phi_k\}$  to belong to the upper class. It generalizes the condition found by Kolmogoroff and Erdös in the special case where  $X_k$  assumes the values  $\pm 1$  only, each with probability 1/2; however, it is different in form. The new theorem also contains a result of Marcinkiewicz and Zygmund on the necessity of the condition imposed by Kolmogoroff on the  $X_k$  in his proof of the law of the iterated logarithm. (Received November 21, 1942.)

### 88. P. G. Hoel: On indices of dispersion.

The sampling distribution of the index of dispersion for binomial and Poisson distributions is investigated by means of semi-invariants. Approximations to terms of order  $N^{-3}$  are obtained for the descriptive moments of the distribution, by means of which the accuracy of the chi-square approximation can be determined. (Received October 30, 1942.)

#### TOPOLOGY

## 89. R. F. Arens: Homeomorphism groups of a space. Preliminary report.

Let A be a locally bicompact, locally connected Hausdorff space, and let G be a group of homeomorphisms of A. Then there is a certain topology N making G into a topological group (Pontrjagin, *Topological groups*, Princeton, 1939). This topology N is the weakest admissible topology that can be introduced into G, in this sense: Sets in G open by N are open by any other admissible topology M. A topology M for a group of homeomorphisms, G, of a space A is called admissible if by using that topology for G the two functions g(a) and  $g^{-1}(a)$ , where  $g \in G$  and  $a \in A$ , become continuous functions of both arguments g and a simultaneously. The topology N is determined by the following system of neighborhoods of the identity in G: Select in A an open set W whose closure is bicompact, and another open set whose closure K is contained in W. Then the set U of all  $g \in G$  which transform K into W is defined to be a neighborhood of the identity. The set of all such U together with all their finite

1943]