VARIATIONAL METHODS FOR THE SOLUTION OF PROBLEMS OF EQUILIBRIUM AND VIBRATIONS

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As Henri Poincaré once remarked, "solution of a mathematical problem" is a phrase of indefinite meaning. Pure mathematicians sometimes are satisfied with showing that the non-existence of a solution implies a logical contradiction, while engineers might consider a numerical result as the only reasonable goal. Such one sided views seem to reflect human limitations rather than objective values. In itself mathematics is an indivisible organism uniting theoretical contemplation and active application.

This address will deal with a topic in which such a synthesis of theoretical and applied mathematics has become particularly convincing. Since Gauss and W. Thompson, the equivalence between boundary value problems of partial differential equations on the one hand and problems of the calculus of variations on the other hand has been a central point in analysis. At first, the theoretical interest in existence proofs dominated and only much later were practical applications envisaged by two physicists, Lord Rayleigh and Walther Ritz; they independently conceived the idea of utilizing this equivalence for numerical calculation of the solutions, by substituting for the variational problems simpler approximating extremum problems in which but a finite number of parameters need be determined. Rayleigh, in his classical work—*Theory of sound*—and in other publications, was the first to use such a procedure. But only the spectacular success of Walther Ritz and its tragic circumstances caught the general interest. In two publications of 1908 and 1909 [**39**], Ritz, conscious of his imminent death from consumption, gave a masterly account of the theory, and at the same time applied his method to the calculation of the nodal lines of vibrating plates, a problem of classical physics that previously had not been satisfactorily treated.

Thus methods emerged which could not fail to attract engineers and physicists; after all, the minimum principles of mechanics are more suggestive than the differential equations. Great successes in applications were soon followed by further progress in the understanding of the theoretical background, and such progress in turn must result in advantages for the applications.

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