ENTIRE FUNCTIONS OF EXPONENTIAL TYPE

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1. **Introduction.** An entire function $f(z)$ is said to be of exponential type if it is of order one and mean type; that is, if for some non-negative c and every positive ϵ there is a number $A(\epsilon)$ such that

$$|f(z)| < A(\epsilon)e^{(c+\epsilon)|z|} \tag{1}$$

for all z. The smallest c which can be used in (1) is called the type of $f(z)$. An alternative definition[1] is

$$c = \limsup_{n\to\infty} |f^{(n)}(z)|^{1/n}; \tag{2}$$

it is immaterial which value of z is used in (2). If (1) holds in a region of the z-plane, for example in an angle, $f(z)$ is said to be of exponential type c in that region.

Functions of exponential type have been extensively studied, both for their own sake and for their applications. I shall discuss here a selection of their properties, chosen to illustrate how the restriction (1) on the growth of a function restricts its behavior in other ways.[2]

2. **Representations.** Various formulas are available for representing functions of exponential type. Some of these representations are useful for deriving results of the kind discussed later in this report; and they are of considerable interest for their own sake.

If $f(z)$ is an entire function satisfying (1), there is a function $\phi(w)$, analytic in $|w| > c$, such that

$$f(z) = \int_C e^{zw}\phi(w)dw, \tag{3}$$

where C is any contour containing $|w| = c$ in its interior.[3] The function $\phi(w)$ is defined by either of the equivalent formulas

$$\phi(w) = \frac{1}{2\pi i}\sum_{n=0}^{\infty}\frac{n!a_n}{w^{n+1}} \quad \text{if} \quad f(z) = \sum_{n=0}^{\infty} a_n z^n$$

and

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[1] See [**10**, p. 241].

[2] For a report on functions of exponential type from another point of view, see [**10**].

[3] See [**25**, pp. 578–586].