described in this brief discussion, it is found that the third integer is the sum of three biquadrates, and gives Norrie's solution

$$
d=198593=\frac{353^{4}-272^{4}}{15^{4}}=21^{4}+8^{4}+2^{4}
$$

The seventh integer furnishes a second numerical solution

$$
d=5086913=\frac{651^{4}-599^{4}}{10^{4}}=43^{4}+34^{4}+24^{4}
$$

It is the intention of the writer to carry the investigation of each case as far as values of $x<1250\left(=2 \cdot 5^{4}\right)$. The fact that an additional solution was obtained in the first case examined would seem to give some promise that other solutions exist among the lower integers.

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# ON THE INVERSION OF THE $q$-SERIES ASSOCIATED WITH JACOBIAN ELLIPTIC FUNCTIONS 

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The elliptic functions sn $(u, k)$, cn $(u, k)$ and $\operatorname{dn}(u, k)$ may be computed from theta functions by well known methods outlined in standard texts. (See, for instance, Whittaker \& Watson, Modern Analysis, 4 th edition, p. 485.) Given the modulus $k$, and $k^{\prime}=\left(1-k^{2}\right)^{1 / 2}$, there is associated with $k, k^{\prime}$ a function $\epsilon$ defined by

$$
\epsilon=\frac{1}{2}\left[\frac{1-\left(k^{\prime}\right)^{1 / 2}}{1+\left(k^{\prime}\right)^{1 / 2}}\right]=\frac{1}{2} \frac{\vartheta_{2}\left(0, q^{4}\right)}{\vartheta_{3}\left(0, q^{4}\right)}
$$

Values of theta functions for a given parameter $q$ can be readily computed, and the Jacobi elliptic functions turn out to be ratios of the theta functions.

The series for $\epsilon$ in terms of $q$ is given by

$$
\begin{equation*}
\epsilon=\sum_{k=0}^{\infty} q^{(2 k+1)^{2}} / 1+2 \sum_{k=1}^{\infty} q^{4 k^{2}} . \tag{1}
\end{equation*}
$$

[^0]
[^0]:    Presented to the Society, February 28, 1942 ; received by the editors December 19, 1941.
    ${ }^{1}$ The above results were obtained in the course of work carried out by the Mathematical Tables Project, conducted by the Work Projects Administration for the City of New York under the sponsorship of the National Bureau of Standards.

