DIFFERENTIAL EQUATIONS WITH GENERAL BOUNDARY CONDITIONS

WILLIAM M. WHYBURN

This paper concerns ordinary differential equations in the real domain. More specifically, it discusses systems of first order, nonsingular, equations of the form

$$\frac{dy_i}{dx} = f_i(x, y_1, y_2, \cdots, y_n), \qquad i = 1, 2, \cdots, n, \tag{1}$$

together with boundary conditions at one, two, or more points of the interval of definition. System (1) is quite general and can be made to include the nonsingular equations of the nth order as special cases. The paper makes no attempt to generalize the equations beyond those that occur in standard treatments. Any elements of generality or novelty introduced appear in connection with the types of boundary conditions discussed. The mathematical literature of the past century contains many results for system (1) with boundary conditions at one point—the so-called fundamental existence theorems occupying central positions—and with boundary conditions at two points of the interval where second order linear systems have been of prime importance. These important bodies of results have been suveyed before the Society through addresses by Bôcher, Bliss, Pell-Wheeler, Reid and others, and therefore are given little mention in the present paper. We discuss results that have been obtained in cases where the boundary conditions apply to n or less points, to any finite number of points, to any infinite point set of the first species,[1] and where the boundary conditions contain integrals over an interval. The literature on differential systems with such boundary conditions is not extensive although many substantial results have been obtained and potential applications exist in a number of fields.

1. **Fundamental existence theorems.** Let the real functions $f_i(x, y_1, \cdots, y_n)$ in system (1) be defined over a domain

$$D: \begin{cases} (a, b): & a \leqq x \leqq b \\ R: & A_i < y_i < B_i \qquad (i = 1, \cdots, n), \end{cases}$$

An address delivered before the Pasadena meeting of the Society on November 22, 1941, by invitation of the Program Committee; received by the editors May 28, 1942.

[1] A set of the first species is one which has at most a finite number of nonvacuous derived sets. Any such set is at most denumerably infinite. See Hobson, *Theory of Functions of a Real Variable*, vol. 1, 2d edition, Cambridge, 1920, pp. 71 and 79.