CLASSES OF MAXIMUM NUMBERS ASSOCIATED WITH TWO SYMMETRIC EQUATIONS

H. A. SIMMONS

1. **Introduction.** Let $\sum_{i,j}(1/x)$ stand for the elementary symmetric function of the jth order of the i reciprocals $(1/x_p)$ $(p=1, 2, \cdots, i>0)$ with

$$\sum_{i,j}(1/x) \equiv 0 \quad \text{when} \quad i<j \text{ or } j<0,$$
$$\equiv 1 \quad \text{when} \quad j=0$$

($\sum_{i,j}(x)$ having a similar meaning for the x_p themselves).

Here we extend the work of papers I,[1] II,[2] III[3] by obtaining relative to equations (1) and (1.1) below results analogous to those in I, II, III

$$\sum_{n,n-1}(1/x) + \sum_{i=1}^{m} a_i[\pi(x)]^{-i} = b/a, \tag{1}$$
$$a = (c+1)b - 1, \ \pi(x) = x_1x_2\cdots x_n,$$

(1)[4]

$$\sum_{n,n-2}(1/x) + \lambda\sum_{n,n-1}(1/x) + \mu\sum_{n,n}(1/x) = b/a; \tag{1.1}$$

in (1), b, c, and m are arbitrary positive integers, $n>1$, and the a_i are any non-negative real numbers; in (1.1), a and b are as in (1), $n>2$, λ is a non-negative integer, and μ is a positive integer.

We have not seen previous mention of (1); the case of (1.1) in which $\mu=0$ was treated in II and that in which $\lambda=\mu=1$ was treated in III. Our procedure for (1) does not suffice for the equation that is obtained by adding to the left member of (1.1) the terms

$$\sum_{i=2}^{m} a_i[\pi(x)]^{-i}.$$

The following definitions and notation from I will be frequently

Presented to the Society, September 5, 1941; received by the editors June 27, 1941.

[1] H. A. Simmons, Transactions of this Society, vol. 34 (1932), pp. 876–907.

[2] Norma Stelford and H. A. Simmons, this Bulletin, vol. 40 (1934), pp. 884–894.

[3] H. A. Simmons and W. E. Block, Duke Mathematical Journal, vol. 2 (1936), pp. 317–340.

[4] In so far as we know, the form of the right member of equation (1) was first used by Tanzo Takenouchi in the Proceedings of the Physico-mathematical Society of Japan, (3), vol. 3 (1921), pp. 78–92.