$\delta_1\omega=\omega^{\rho_1+1}$, and $\delta_1 j=\omega^{\rho_1}v_1 j+\omega^{\rho_2}v_2+\cdots+\omega^{\rho_z}v_z<\omega^{\rho_1}(v_1 j+1)$;

$$\sigma(\delta_1\mu,\ \delta_1 j)<\sigma(\delta_1\mu,\omega^{\rho_1}(v_1 j+1))<\delta_1\mu+\omega^{\rho_1+1}=\delta_1\mu+\delta_1\omega.$$

By (2), $\pi(\delta^\mu,\ \delta^j)<\omega^{\delta_1\mu+\delta_1\omega}=(\omega^{\delta_1})^{(\mu+\omega)}\leqq\delta^{\mu+\omega}\leqq\delta^\delta$.

Hence by (1), the order type of S is less than $\pi(\omega^\delta,\ \delta^\delta)$. This is a contradiction since S was the segment of M^δ of order type $\pi(\omega^\delta,\ \delta^\delta)$.

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A CHARACTERIZATION OF ABSOLUTE NEIGHBORHOOD RETRACTS

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By an *absolute neighborhood retract* (ANR) I mean a separable metrizable space which is a neighborhood retract of every separable metrizable space which contains it and in which it is closed. This generalization of Borsuk's original definition[1] was given by Kuratowski[2] for the purpose of enlarging the class of absolute neighborhood retracts to include certain spaces which are not compact. The space originally designated by Borsuk as absolute neighborhood retracts (or $\mathfrak{R}$-sets) will now be referred to as compact absolute neighborhood retracts. Many of the properties of compact ANR-sets hold equally for the more general ANR-sets.[3]

The Hilbert parallelotope Q, that is, the product of the closed unit interval $[0, 1]$ with itself a countable number of times is a "universal" compact ANR in the sense that[4] every compact ANR is homeomorphic to a neighborhood retract of Q. The classical theory of Borsuk makes good use of the imbedding of compact ANR-sets in Q. The problem solved here is that of finding a "universal" ANR.

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[1] Fundamenta Mathematicae, vol. 19 (1932), pp. 220–242.

[2] Fundamenta Mathematicae, vol. 24 (1935), p. 270, Footnote 1.

[3] Ibid., pp. 272, 276, and 277, and Footnote 1, p. 279 and Footnote 3. Note that Theorem 12, Fundamenta Mathematicae, vol. 19 (1932), p. 229, is not true for general ANR-sets. In fact let $A=\sum S_n$ where S_n is the plane circle of radius 2^{-n} and center $(3\cdot 2^{-n},\ 0)$; let $f(x,\ y)=(x,\ |y|)$ for $(x,\ y)\in A$ and let

$$f_n(x,\ y)=\begin{cases}(x,\ |y|), & \text{for } (x,\ y)\in A-S_n,\\ (x,\ y), & \text{for } (x,\ y)\in S_n.\end{cases}$$

Then $f_n\to f$ in A^A; f can be extended to the half-plane $\{x>0\}$, but none of the maps f_n can. A is an ANR-set. Theorem 16, Fundamenta Mathematicae, vol. 19 (1932), p. 230, is also false for general ANR-sets.

[4] Fundamenta Mathematicae, vol. 19 (1932), p. 223.