tive radius r. Let the center x_0 be the sequence $\{k_i^0\}$, and let s be chosen so large that $2^{-s-1}+2^{-s-2}+\cdots<r$. Now a point $X\equiv\{j_i\}$ of D exists such that $\lim_n f_n(X)=+\infty$, and such that $j_{s+1}>k_s^0$. If we define x_1 as $(k_1^0, k_2^0, \cdots, k_s^0, j_{s+1}, j_{s+2}, \cdots)$, then x_1 belongs to K and $\lim_n f_n(x_1)=+\infty$. Consequently x_1 cannot be a point of U_μ and this contradiction establishes U as a set of the first category.

In a similar fashion it may be shown that the set V of all x in D for which $\lim\inf_n f_n(x)>-\infty$ is likewise a set of the first category. Hence if we set $W\equiv U+V$ the theorem follows.

Finally, let $\sum u_k$ be a convergent series of complex terms for which $\sum|u_k|=+\infty$, and for this series let $\phi_n(\xi)$ $[f_n(x)]$ be defined as in (1.5) [(2.4)]. We may consider the series of real and imaginary parts in the light of Theorem 2 [Theorem 3] and thus show that the set of all ξ on I [x in D] for which we have $\lim\sup_n|\phi_n(\xi)|<\infty$ [$\lim\sup_n|f_n(x)|<\infty$] is a set of the first category.

Michigan State College

A FORMULA FOR THE DIRECT PRODUCT OF CROSSED PRODUCT ALGEBRAS

SAUNDERS MacLANE AND O. F. G. SCHILLING

1. **Introduction.** In this note we wish to present a uniform treatment of certain properties of crossed products. A crossed product over any field F is an algebra determined by a finite, separable, normal extension N of F, with a Galois group Γ, and a certain factor set[1] $\boldsymbol{h}$ of elements $h_{S,T}$ in N, for automorphisms S and T in Γ. The crossed product $(N, \Gamma, \boldsymbol{h})$ consists of all sums $\sum u_S z_S$, where the coefficients z_S lie in N, and the fixed elements u_S have the multiplication table

$$u_S u_T = u_{ST} h_{S,T}, \qquad z u_S = u_S z^S, \qquad z \text{ in } N. \tag{1}$$

Let K be a normal subfield of N, corresponding to the subgroup Δ of the Galois group Γ. A factor set $\boldsymbol{g}$ in N is called *symmetric* in Δ if $g_{S,T}=g_{U,V}$ whenever SU^{-1} and TV^{-1} are in Δ.

Presented to the Society, May 3, 1941; received by the editors March 31, 1941.

[1] Definitions are given in A. A. Albert, *Structure of Algebras*, American Mathematical Society Colloquium Publications, vol. 24, 1939. Theorems cited below without explicit source all refer to this work.