fined by pairs of functions of two complex variables, the pseudo-conformal group G. This is characterized by the preservation of Kasner's pseudo-angle between a curve and a hypersurface at their common point. Any system of ∞^3 curves which is pseudo-conformally equivalent to a parallel set of lines is said to be bi-isothermal. Any such system consists of ∞^2 isothermal families, each family living on a conformal surface. A characteristic property of bi-isothermal systems is that the pseudo-angle between this system and any parallel pencil of hyperplanes is a biharmonic function. Finally G is characterized within the group of point transformations by the preservation of the totality of all bi-isothermal systems. (Received September 29, 1941.)

481. Walter Prenowitz: *Descriptive geometries as multigroups.*

Let G be the set of points of a descriptive space of arbitrary dimension. In G, define $a+b$ as the set of points between a and b if $a\neq b$, and $a+a$ as a. Then G becomes an abelian multigroup of special type. Convex sets and linear manifolds appear respectively as semigroups (subsets closed under $+$) and subgroups of G. Half-spaces (rays, half-planes, and so on) are cosets, when the latter are properly defined, and some of the elementary properties of half-spaces follow from a coset decomposition theorem. Elementary properties of these three types of figures are derived algebraically and many familiar group theoretic concepts as factor group, homomorphism, congruence relation are used in the development. (Received September 29, 1941.)

482. C. V. Robinson: *Spherical theorems of Helly type and congruence indices of spherical caps.*

The theorems mentioned in an earlier abstract (47-1-67) have been extended to the n-sphere. The principal theorem of Helly type reads: If each $n+k+2$ members of a family of convex subsets of the n-sphere intersect and if one member contains no k-dimensional great hypersphere then there is a point common to all the sets of the family. A study is also made of the congruence indices of spherical caps of various radii with respect to the class of semi-metric spaces. For example, it is found that a cap of spherical radius $\rho<\pi r/2$ of the 2-sphere of radius r will contain isometrically any semi-metric space of more than 6 points if every quadruple of the space is isometrically imbeddable in the cap, that is, the cap has the congruence indices [4, 2]. (Received September 23, 1941.)

STATISTICS AND PROBABILITY

483. J. F. Daly: *A problem in estimation.*

Consider a normal population in which each individual is characterized by the variates $y_1, \cdots, y_p, y_{p+1}, y_{p+2}$. Suppose that the latter two are not directly observable, but that for given values of y_{p+1}, y_{p+2} the first set of y's is independently distributed about the "regression line" $y_k=y_{p+1}+ky_{p+2}$ $(k=1, \cdots, p)$ with a common variance σ^2. For each individual, one can thus determine values $\hat{y}_{p+1}, \hat{y}_{p+2}$ from the observed $y_1, \cdots, y_p$, using the method of least squares. Assuming a similar relation between the expected values of $y_1, \cdots, y_{p+2}$ in the original population, these estimates $\hat{y}_{p+1}, \hat{y}_{p+2}$ are of course unbiased. However, if we calculate these $\hat{y}$'s for each individual of a sample of N and substitute them in the Pearson product-moment correlation formula the estimate of the correlation between y_{p+1} and y_{p+2} thus obtained is somewhat biased. The bias depends on the number of observable y's and on the size of the variances and covariances of y_{p+1}, y_{p+2} relative to σ^2. (Received September 2, 1941.)