# INDECOMPOSABLE CONNEXES[1]

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DEFINITION. *A connected set $M$ is an indecomposable connexe if and only if, for every two connected subsets $H$ and $K$ of $M$ such that $M=H+K$, either $H$ and $M$ or $K$ and $M$ have the same closure.*[2]

Any connected subset $N$ of an indecomposable continuum W, which is dense in $W$, such as any set of composants of $W$ or $W$ itself, is an indecomposable connexe, as is also a widely connected set.[3]

EXAMPLE A.[4] Let, in a euclidean plane, $U$ be the points of a square, $Q$, plus its interior. Let $U_i$ $(i=1, 2, 3, \cdots)$ be a set of mutually exclusive arcs each contained in $U$ and having one and only one point, an end point, common with $Q$. Let the $U_i$'s be taken so that every plane region of $U$ is joined to every linear region of $Q$ by at least one $U_i$. Let $M=U-(U_1+U_2+\cdots)$. Then $M$ is connected[5] and such that, if $H$ and $K$ are connected and their sum is $M$, either $H$ and $M$ or $K$ and $M$ have the same closure. Hence $M$ is an indecomposable connexe.

EXAMPLE B. Let, in a euclidean plane, $U$ be the points of a triangle plus its interior, one vertex of which is the point $a$. Let $U_i$ $(i=1, 2, 3, \cdots)$ be a set of arcs, mutually exclusive, except for having the common end point $a$, and whose sum is dense in $U$. Let further the $U_i$'s be taken so that each two plane regions of $U$ are joined by at least one $U_i$. Let $M=U-(U_1+U_2+\cdots)$. It can be shown without difficulty that $M$ is an indecomposable connexe.

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[1] Presented to the Society November 23, 1940.

[2] See S. Eilenberg, *Topology du plan*, Fundamenta Mathematicae, vol. 26, p. 81, for a definition of an indecomposable connected space. This definition is seen to be equivalent to the above for the types of spaces considered in these two papers.

[3] For definition and example see P. M. Swingle, *Two types of connected sets*, this Bulletin, vol. 37 (1931), pp. 254–258.

[4] E. W. Miller communicated this interesting example to me by letter in 1937 calling attention to its relation to a widely connected set. The method of construction is somewhat similar to the well known boring process used to obtain a plane indecomposable continuum. See K. Yoneyama, *Theory of continuous sets of points*, Tôhoku Mathematical Journal, vol. 12 (1917), p. 60. That either $H$ and $M$ or $K$ and $M$ have the same closure is seen above by supposing that neither $H$ nor $K$ is dense in $M$, from which it readily follows that $H$ and $K$ can each have at most one point common with $Q$ itself.

[5] E. W. Miller, *Some theorems on continua*, this Bulletin, vol. 46 (1940), p. 153, Theorem 3.