THE ACYCLIC ELEMENTS OF A PEANO SPACE

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We suppose throughout that S is a Peano space. The notion of a cyclic element of such a space was introduced by G. T. Whyburn[1] and A-sets were introduced independently by W. L. Ayres[2] and Whyburn. Our present purpose is to consider a class of sets which may be regarded as the duals of cyclic elements.

Let $Q(S)$ denote the set of all cut-points and end-points of S.[3] Then each component of $Q(S)$ will be called an *acyclic element* of S. If p and q are two points of S then we write $p \sim q$ to mean that *no point separates p from q in S.* A set will be termed *acyclic* if it contains no simple closed curve, *cyclic* if each pair of points is on a simple closed curve of the set.

(i) *Each cyclic element [acyclic element] is a cyclic [acyclic] A-set.*

Proof. We prove only the second statement. Let F be an acyclic element, $x_n \in F$ and suppose that $x_n \rightarrow x$. If x is not in F then since $F+x$ is connected we must have $x \in S-Q(S)$, from the definition of F as a component of $Q(S)$. Consequently x is a point of a cyclic element E of S. If $(F+x)\cdot E$ contained only the point x then it would follow that F was contained in a single complementary domain R of E and thus $x=F(R)=\overline{R}-R$. Consequently x would be a cut-point, which is obviously impossible. We conclude that $(F+x)\cdot E$ is a nondegenerate connected set. It is clear that no point of this set is an end-point and hence the set contains uncountably many cut-points. But no cyclic element contains more than a countable number of cut-points. We conclude that $x \in F$ and hence F is closed. If F is not an A-set there is an arc pxq which meets F in the set $p+q$. It is manifest that no point separates p from q in S since F is connected. That is, $p \sim q$ and hence $p+q \subset E$, a true cyclic element. By the argument given

[1] See Kuratowski and Whyburn, Fundamenta Mathematicae, vol. 16 (1930), p. 305. By a cyclic element we understand a *nondegenerate* set such that any two points lie on a simple closed curve and which is maximal relative to this property. This is not the definition given by Kuratowski and Whyburn but is equivalent (cf. G. T. Whyburn, this Bulletin, vol. 38 (1931), p. 429 and references given there) and is in fact Whyburn's original definition. It seems more natural in the present setting.

[2] See W. L. Ayres, this Bulletin, vol. 46 (1940), p. 794, for references. An A-set is a *closed* arc set.

[3] The terms *cut-point* and *end-point* without qualification refer to the space S. The proofs of all the statements concerning cyclic elements will be found in Kuratowski and Whyburn. We assume considerable familiarity with this fundamental work.