CONDUCTION OF HEAT IN REGIONS BOUNDED BY PLANES AND CYLINDERS

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1. **Introduction.** The Green's functions for regions bounded by surfaces of the cylindrical coordinate system are well known.[1] From them solutions may be obtained for problems in which the initial temperature is zero and the surfaces are kept at temperatures which are known functions of the coordinates; the application of Green's function in regions extending to infinity has not been completely studied and the conditions to be satisfied by functions prescribed in such regions are not known. An alternative method of solving such problems consists of using the Laplace transformation and solving the resulting subsidiary equation by separation of variables; here it is found necessary to assume that temperatures given on surfaces extending to infinity satisfy very narrow conditions, such as those of Fourier's or Weber's integral theorems. This is illustrated in §3. Problems of conduction of heat in regions bounded by cylinders and planes, some of which extend to infinity, and with constant surface (and initial) temperatures, or with a radiation boundary condition

$$\frac{\partial v}{\partial n} + h(v - v_0) = 0$$

at a surface, are of considerable importance and the constant surface temperatures do not satisfy the conditions referred to above [cf. §3]. The method given below gives a simple solution of all such problems; the results given form a complete set from which the solutions of all temperature problems in solids bounded by a cylinder and planes perpendicular to its axis, with constant surface (and initial) temperatures, can be written down. Problems involving a radiation boundary condition at some of the surfaces, and problems on the hollow cylinder, may be solved in the same way.

The method was suggested by that given in a previous note[2] which consisted of the use of a double Laplace transformation; this gives a solution under very wide conditions on the surface temperatures, roughly that they be of exponential type in the space variable. This method also allows verification that the solution obtained does satisfy

[1] Carslaw, *Conduction of Heat* (2d edition, 1921) chaps. 9 and 10; for a discussion using the Laplace transformation see Carslaw and Jaeger, Journal of the London Mathematical Society, vol. 15 (1940), p. 273.

[2] This Bulletin, vol. 46 (1940), pp. 687–693. This paper will be referred to as I.