than one. The implications of the non-existence of conjugate points are also investigated in the case of surfaces of the topological type of the torus. It is found that to a large extent the geodesics have the properties of those on a manifold for which the Gaussian curvature vanishes identically. If an added hypothesis concerning the non-existence of focal points is invoked, it can be shown that the curvature does vanish identically. (Received June 7, 1941.)

426. Abraham Schwartz: *A consequence of the Ricci equations for Riemannian manifolds.*

The paper considers the normal spaces at a point of a riemannian manifold of m dimensions which is imbedded in a manifold of constant curvature of n dimensions, $V_m \subset S_n$. With each normal space, $x=1, \cdots, k$, there is associated a curvature tensor $H(x)$ in a well known way, each tensor $H(x)$ being related to the preceding one, $H(x-1)$, by a Ricci equation. From these Ricci relationships information concerning the dimensionalities of the various normal spaces can be deduced. For instance, if the first normal space is one-dimensional, then the second normal space is zero or one-dimensional according as the rank of the second fundamental form is $\geqq 2$ or <2; if the first normal space is two-dimensional, then the second normal space is zero-, one-, or two-dimensional depending on the nature of the elementary divisors of the two second fundamental forms. Analogous theorems are true for the other normal spaces. (Received July 24, 1941.)

427. S. M. Ulam and D. H. Hyers: *On approximate isometries.* Preliminary report.

Let E and F be metric spaces and let ϵ be a positive number. The question studied is the following. If $T(x)$ is a continuous transformation of E into F such that $|\rho(T(x), T(y)) - \rho(x, y)| \leqq \epsilon$ for all x and y, then does there exist an isometric transformation $S(x)$ of E into F such that for all, x, $\rho(S(x), T(x)) \leqq k\epsilon$, where k is a positive constant, depending only on the spaces E and F? The question is answered in the affirmative for the cases in which E and F are Hilbert spaces or finitely dimensional euclidean spaces. (Received July 30, 1941.)

Logic and Foundations

428. Garrett Birkhoff: *Metric foundations of geometry.*

New derivations of fundamental theorems of euclidean, hyperbolic and spherical geometry, with particular reference to the lattice of subspaces, are given. The postulates are: (1) metric postulates of Fréchet, (2) straight line postulates of Menger, in a weakened form valid in any riemannian geometry, (3) the postulate of Pasch. Thus the local compactness postulate of Busemann's system is dispensed with, and his straight line postulates are weakened; on the other hand, the postulate of Pasch is not restricted to 3 points. The proofs are elementary throughout. (Received July 7, 1941.)

429. Henry Blumberg: *A reconsideration of the paradoxes in the theory of sets.*

Proceeding from an intuitive bias that it should be possible to eliminate the paradoxes in question along the line of natural human understanding, without reforming our familiar logic or invoking ingenious technical or professional stratagems, the