ON THE MAPPING OF THE SETS OF 24 POINTS OF THE SYMMETRIC SUBSTITUTION GROUP G_{24} IN ORDINARY SPACE UPON A HYPERQUADRIC CONE

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Introduction. The mapping of the sextuples of the symmetric substitution group G_6 in a plane upon a quadric has been done by Emch.[1] The 24 permutations of 4 elements x_1, x_2, x_3, x_4 considered as projective coordinates in ordinary space determine a configuration[2] which may be mapped on a hypersurface in S_4. I shall show that the hypersurface on which we will map is a hyperquadric cone. The map of every configuration on the hyperquadric will be a configuration in ordinary space, invariant under the G_{24}.

The mapping of the G_{24}. We shall represent the elementary symmetric functions as follows:

$$\begin{aligned}
\phi_1 &= x_1 + x_2 + x_3 + x_4,\\
\phi_2 &= x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4,\\
\phi_3 &= x_1x_2x_3 + x_1x_3x_4 + x_1x_2x_4 + x_2x_3x_4,\\
\phi_4 &= x_1x_2x_3x_4.
\end{aligned}$$

Let $y_i = A_i\phi_1^4 + B_i\phi_1^2\phi_2 + C_i\phi_2^2 + D_i\phi_1\phi_3 + E_i\phi_4$ where $i = 1, 2, 3, 4, 5$. There are five linearly independent y's. We shall consider the y's as the coordinates of a point in S_4. Thus to each point in (x), and consequently to each of 24 points in (x), corresponds a point (y) in S_4. The locus of the points (y) is a hypersurface of some order in S_4.

Let us choose five linearly independent y's. (For every choice of y's we will get some hypersurface and all these hypersurfaces will be linearly related.)

$$\begin{aligned}
\rho y_1 &= \sum x_1^4 = \phi_1^4 - 4\phi_1^2\phi_2 + 2\phi_2^2 + 4\phi_1\phi_3 - 4\phi_4,\\
\rho y_2 &= \sum x_1^2x_2^2 = \phi_2^2 - 2\phi_1\phi_3 + 2\phi_4,\\
\rho y_3 &= \sum x_1^3x_2 = \phi_1^2\phi_2 - 2\phi_2^2 - \phi_1\phi_3 + 4\phi_4,\\
\rho y_4 &= \sum x_1^2x_2x_3 = \phi_1\phi_3 - 4\phi_4,\\
\rho y_5 &= \sum x_1x_2x_3x_4 = \phi_4.
\end{aligned}$$

If we eliminate the ϕ's we get a hyperquadric cone Q given by

[1] This Bulletin, vol. 33 (1927), pp. 745–750.

[2] Veronese Annali di Mathematica, (2), vol. 2, p. 93.