

rem that ( $p/q$  being irreducible)

$$p = p_n + \epsilon p_{n-1}, \quad q = q_n + \epsilon q_{n-1}, \quad \epsilon = \pm 1,$$

for the convergents to  $x$  are all  $[e/o]$  or  $[o/e]$ . Write  $X = [a_{n+1}, a_{n+2}, \dots]$ ,  $Y = [a_n, a_{n-1}, \dots, a_2]$ . Then if  $n \geq 2$ ,

$$\theta = \frac{(Y + \epsilon)(X - \epsilon)}{XY + 1} = 1 - \frac{2 - \epsilon(X - Y)}{XY + 1} > 1 - \frac{2 + X + Y}{XY + 1},$$

$$\begin{aligned} XY + 1 - E(2 + X + Y) &= (X - E)(Y - E) - E^2 - 2E + 1 \\ &> (E + 1)^2 - E^2 - 2E + 1 > 0, \end{aligned}$$

$$\theta > 1 - 1/E.$$

If  $n = 1$ , then  $p = p_1 + 1$ ,  $q = q_1 = 1$ ,  $\theta = 1 - [0, a_2, \dots] > 1 - 1/E$ .

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## MEASURABILITY AND DISTRIBUTIVITY IN THE THEORY OF LATTICES<sup>1</sup>

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**Introduction.** Garrett Birkhoff<sup>2</sup> derived the following self-dual symmetric condition that a metric lattice be distributive:

$$(1) \quad 2[\mu(a \cup b \cup c) - \mu(a \cap b \cap c)] = \mu(a \cup b) - \mu(a \cap b) + \mu(a \cup c) - \mu(a \cap c) + \mu(b \cup c) - \mu(b \cap c).$$

In a previous note<sup>3</sup> the author introduced and discussed a generalization of Carathéodory's notion of measurability<sup>4</sup> with respect to an outer measure function  $\mu$  which applies to arbitrary lattices  $L$ . The  $\mu$ -measurable elements form a subset  $L(\mu)$  consisting of those elements  $a \in L$  which satisfy

$$(2) \quad \mu(a \cup b) + \mu(a \cap b) = \mu(a) + \mu(b)$$

for every  $b \in L$ . Closure properties of  $L(\mu)$  were investigated. In par-

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<sup>2</sup> *Lattice Theory*, American Mathematical Society Colloquium Publications, vol. 25, p. 81. We shall adopt the notation and terminology of this work and shall indicate specific references to it by B.

<sup>3</sup> *A note on measure functions in a lattice*, this Bulletin, vol. 46 (1940), pp. 239-241. We shall indicate references to this paper by M.

<sup>4</sup> *Vorlesungen über Reelle Funktionen*, 2d edition, p. 246.