CREMONA INVOLUTIONS DETERMINED BY TWO LINE CONGRUENCES

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1. **Introduction.** Recently, in this Bulletin,[1] we discussed a multiple null-correspondence formed by two line congruences. The *first congruence* consisted in the lines intersecting a fixed twisted curve of order m and also its $(m-1)$-secant d, and the *second congruence* consisted in the lines intersecting another twisted curve δ_n', of order n, and its $(n-1)$-secant d'.

When $m=2$, the first curve is a conic c_2 having one point on d; a series of space Cremona involutions may now be defined as follows:

A generic point P determines a ray ρ_1 of the first congruence and a ray ρ' of the second congruence. In the null-plane of P, formed by ρ_1 and ρ', lies another ray ρ_2 of the first congruence which intersects ρ' in P', the correspondent of P in the involution.

2. **The defining curves in general position.** Let the equations of d be $x_1=0$, $x_2=0$; and those of d' be $x_3=0$, $x_4=0$. Let the parametric equations of c_2 and of δ_n' be, respectively,

$$\begin{aligned} x_1 &= \lambda^2, & x_1 &= f_n(s, t), \\ x_2 &= \lambda(\lambda - \mu), & x_2 &= F_n(s, t), \\ x_3 &= \lambda(\lambda - \mu), & x_3 &= \prod_1^{n-1} (t_is - s_it)(as + bt), \\ x_4 &= \mu^2, & x_4 &= \prod_1^{n-1} (t_is - s_it)(cs + dt), \end{aligned}$$

where s_i, t_i are the values of the parameters of δ_n' at the $n-1$ points on d'.

Then the equations of the involution are

$$\begin{aligned} x_1' &= (x_2 - x_3)(x_1F - x_2f)^2, \\ x_2' &= (x_1F - x_2f)L, \\ x_3' &= x_3\{(x_2 - x_3)(x_1F - x_2f)(F - kHx_2) + (kHx_1 - f)L\}, \\ x_4' &= x_4\{(x_2 - x_3)(x_1F - x_2f)(F - kHx_2) + (kHx_1 - f)L\}, \end{aligned}$$

where $f\equiv f_n(dx_3-bx_4,\ ax_4-cx_3)$, $k\equiv(ad-bc)$, $F\equiv F_n(dx_3-bx_4,$

[1] E. J. Purcell, *A multiple null-correspondence and a space Cremona involution of order* $2n-1$, this Bulletin, vol. 46 (1940), pp. 339–344.